Symmetry Features and Group Hierarchy Model of Human Symmetry Perception

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Abstract

We used timed trials to assess subjects' ability to perceive various features of two-dimensional symmetry. A model of pattern recognition is proposed to explain subjects' ability to distinguish between wallpapers, images with distinct sets of symmetries. As these wallpapers This model suggests the group-theoretic analysis is potentially parallel to the perception of symmetry. Further, it confirms that despite some skeptics, all types of symmetry seem to be readily perceived by humans. Nonetheless, there are differences in how easily different symmetries are recognized.

Reflection

Figure : Characterized by the number and location of the axes, left: T_1, right: T_1, T_2, D_1, D_2

Glide Reflection

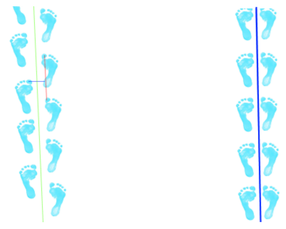


Figure : Characterized by the number and location of the axes, left: T_1 glide, right: T_1 reflection

Symmetry and Wallpapers

Four fundamental types of *Symmetry*: Reflection, Glide Reflection, Rotation, Translation

Wallpapers: Images formed by a combination of Translation and a set of other symmetries

17 distinct *Wallpaper Groups*: each group is a unique combination

Wallpaper groups form a hierarchy

All have translation symmetry; the *tile* is the repeating part

Rotation

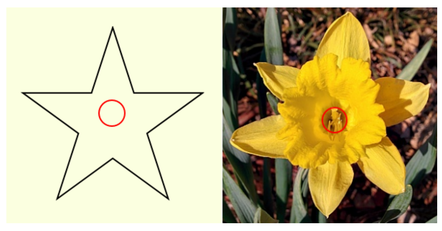


Figure : Characterized by the rotation angles: left: 5-fold, right: 6-fold

Translation

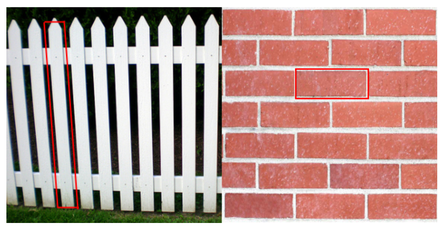


Figure : Characterized by the repeating shape, left: pentagon, right: rectangle

Experiment

106 subjects from Amazon Mechanical Turk

Five seconds to choose before counted as wrong

Choose Image on right that is most similar to Image on left

One Image on right is same group, one is different group

If the images from different have same value for a feature, it is encoded as True

Can examine which features affect accuracy directly

Wallpaper Groups

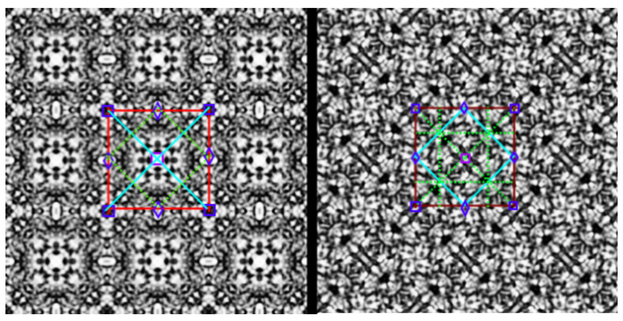


Figure : Different Groups: P4M vs. P4G

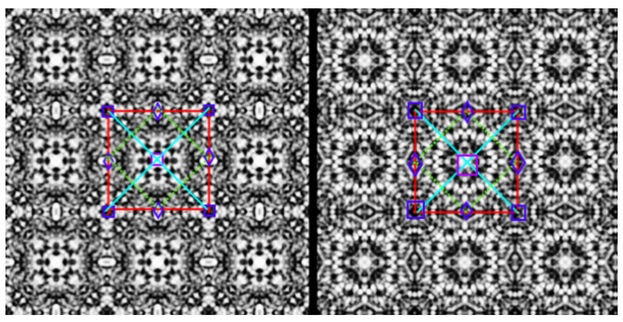


Figure : Same Group: Both P4M.

Subgroup Distance

Distance refers to the *shortest-path* distance between two groups in the Wallpaper Hierarchy. The hierarchy forms the group-theoretic analysis of Symmetry. Distance is thus a measure of this

There could be other ways to measure distance. For instance, an edit distance between the two sets. While the other features are boolean, distance is an integer.

GLMM Fixed Effects

	Est.	SE	z-val	P($<\|z\|$)
(Intercept)	0.960	0.100	9.645	$<$ **0.001***
T1	-0.248	0.042	-5.975	$<$ **0.001***
T2	-0.078	0.040	-1.924	0.0544
D1	-0.314	0.041	-7.732	$<$ **0.001***
D2	0.102	0.044	2.301	0.0214*
2fold	0.012	0.035	0.343	0.7316
3fold	-0.181	0.042	-4.302	$<$ **0.001***
4fold	0.324	0.043	7.555	$<$ **0.001***
6fold	-0.075	0.045	-1.711	0.0870
tile	-0.079	0.056	-1.420	0.1555
distance	0.224	0.019	11.978	$<$ **0.001***

Table : Logistic Linear Mixed Effects model predicting accuracy, with random effects grouped by Participant and Task ID

Wallpaper Hierarchy

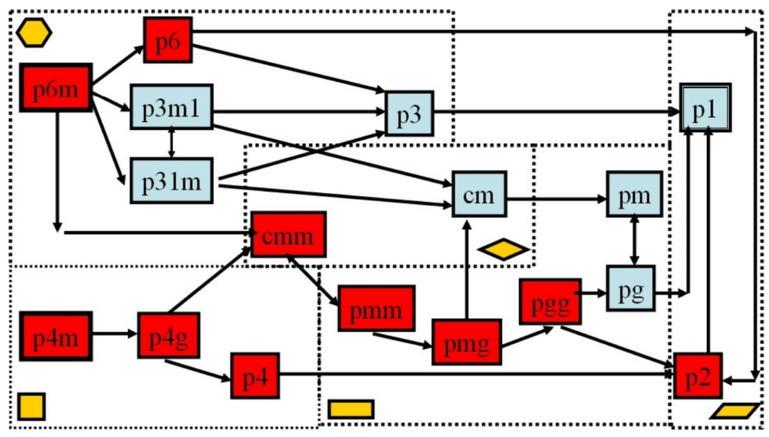


Figure : If an arrow points from any given box, ***A***, toward any given box ***B***, that means that ***B***'s symmetries are subset of ***A***'s symmetries.

Wallpaper Group Symmetries

Group	2-fold	3-fold	4-fold	6-fold	T_1	T_2	D_1	D_2	tile
P1	F	F	F	F	None	None	None	None	O
P2	T	F	F	F	None	None	None	None	O
PM	F	F	F	F	Refl	None	None	None	Re
PG	F	F	F	F	Glide	None	None	None	Re
CM	F	F	F	F	None	None	Refl	None	Rh
PMM	T	F	F	F	Glide	Refl	None	None	Re
PMG	T	F	F	F	Glide	Refl	None	None	Re
PGG	T	F	F	F	Glide	Glide	None	None	Re
CMM	T	F	F	F	None	None	Refl	Refl	Rh
P4	T	F	T	F	None	None	None	None	S
P4M	T	F	T	F	Refl	Refl	Refl	Refl	S
P4G	T	F	T	F	Glide	Glide	Refl	Refl	S
P3	F	T	F	F	None	None	None	None	H
P3M1	F	T	F	F	None	None	Refl	None	H
P31M	F	T	F	F	Refl	Refl	Refl	None	H
P6	T	T	F	T	Refl	None	None	None	H
P6M	T	T	F	T	Refl	Refl	Refl	Refl	H

Table : Rotation symmetry, Reflection Axes, and Tile Shape

Most Difficult Choice

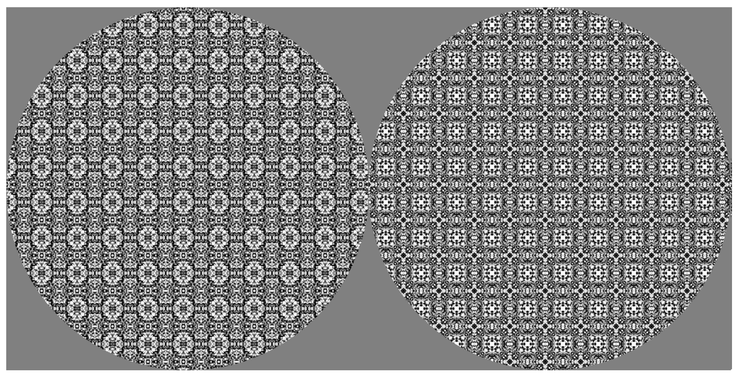
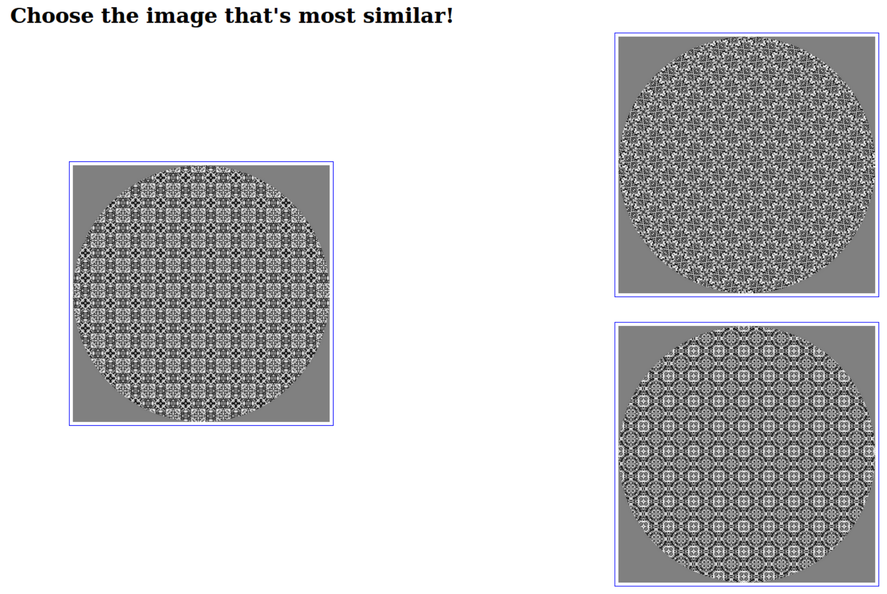


Figure : PMM on the left, P4M on the right.

Task Screenshot



Pairwise Accuracies

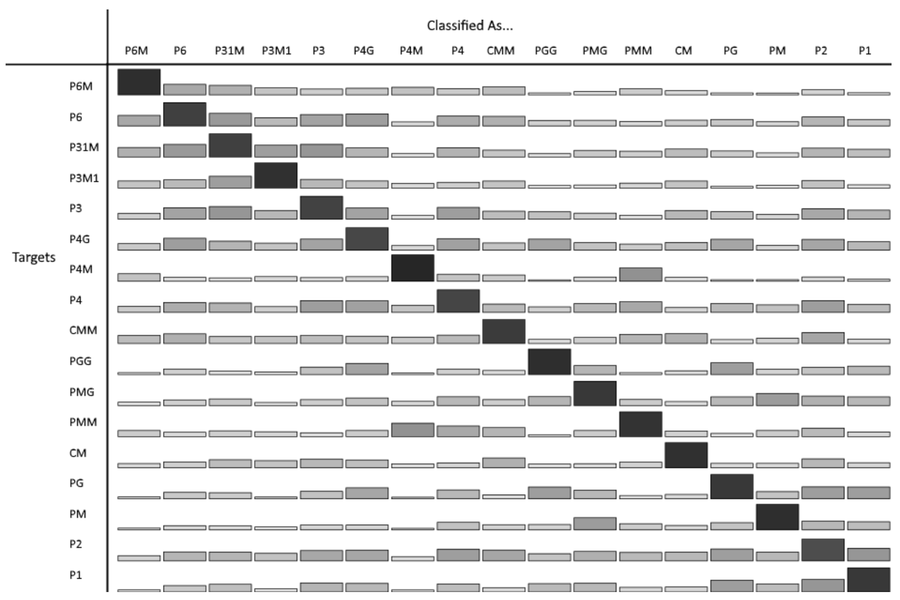


Figure : Main diagonal: overall accuracy, Off-Diagonal: error rates

Conclusions

The best model (by AIC) included subgroup distance, the T_1 axis (lateral reflection symmetry), the D_1 axis (the positive diagonal), 4-fold and 3-fold rotation. Our participants were quite good at distinguishing among the groups, though accuracy varied. *Subgroup distance* was a significant and improved the AIC of every model. This could mean that human pattern analysis at the rapid heuristic level is similar to the mathematical level. Further research could involve using neuroscience methods to draw similar conclusions