Multi-Scale Kernel Operators for Reflection and Rotation Symmetry

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Symmetry

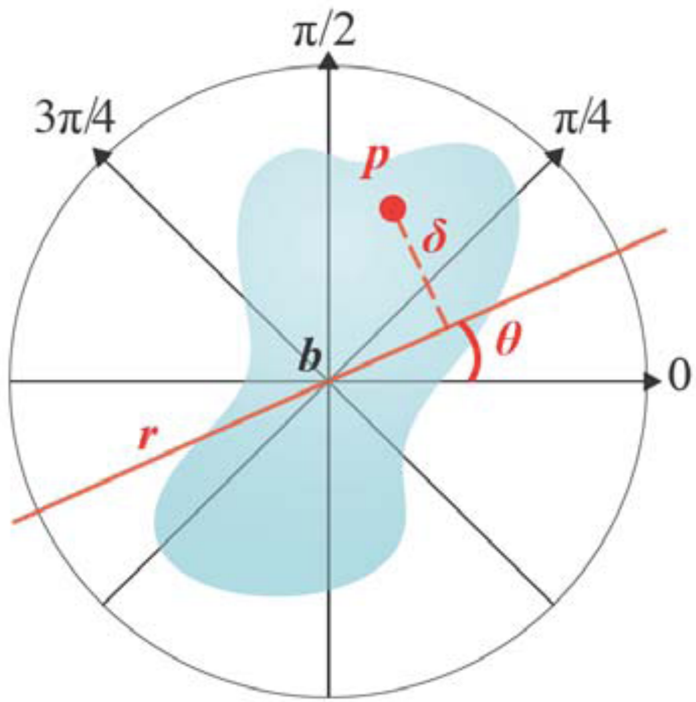
CVPR

- The property of being symmetrical: correspondence in size, shape, and relative position of parts on opposite sides of a dividing line or median plane or about a center or axis.
- In particular, we deal with bilateral symmetry.
- A measure obtained by using correlation with the flipped image around a particular axis.
- Di Gesù et al. (2007) has proven that, in any direction, the optimal symmetry axis corresponds to the maximal correlation of a pattern with its symmetric version.

Symmetry Transform

$$S_\theta(X) = \int_X m(x) \times \delta^2(x, r(b,\theta))\, dx$$

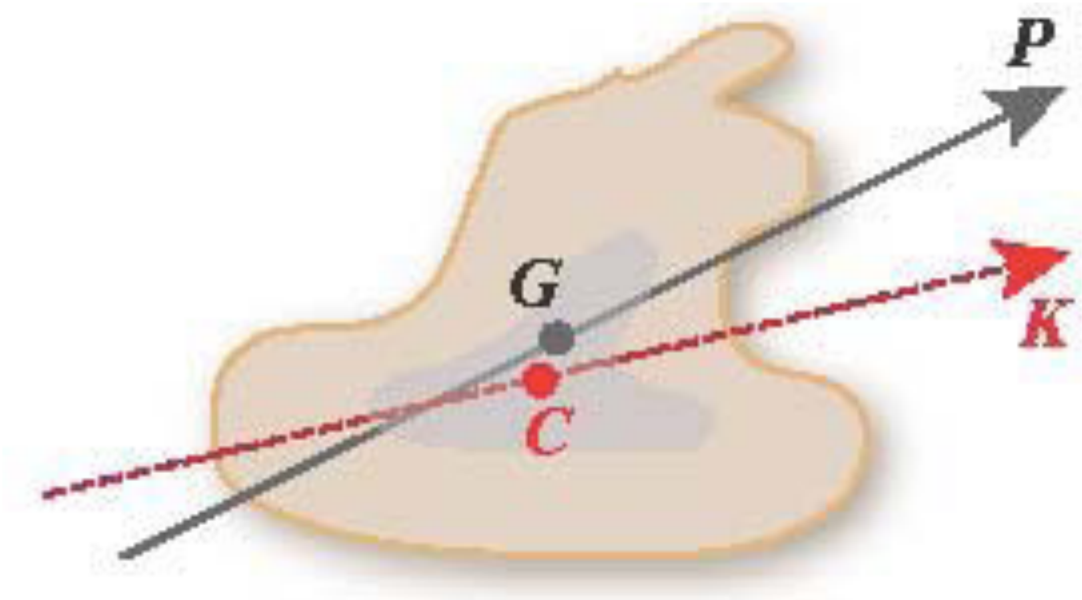
where *r(b,θ)* is the straight line with slope θ passing through b, *m(x)* is the intensity value in $x \in X$, and δ is a distance function of *x* from the straight line.



Symmetry Kernel

- **Definition**

  The S-kernel of the pattern X is the maximal for inclusion symmetric (pattern) subset of X.



Symmetry Kernel

Algorithm

Find the maximum correlation of the picture in a given direction with its mirror symmetric version in that direction.

Foreach $n \times n$ patch X around a pixel i do

  Foreach θ do

    1. Whiten X
    2. Create X_θ as rotated image patch by θ
    3. Create X^x and X^y as reflected patches with respect to x-axis and to y-axis
    4. Calculate the maximum between $X_\theta \otimes X^x$ and $X_\theta \otimes X^y$

  End

$$\hat{\theta} = \arg\max_\theta S_\theta(X)$$

End

Tweaks

- Instead of taking every point in the image, downsample to increase speed by filtering with circular steerable filters (Simoncelli et al., 1992)
- Reflecting the patch around both *x*-axis and *y*-axis will save half the rotations of the patch.
- For color images, RGB space is used and the patch is reflected with respect to the three bands before doing the correlation.

Detecting Multiple Reflection Symmetry

Algorithm

1. Let $p(\theta)$ be the distribution of angles θ (symmetry axis)

2. Create $A = \{\theta \mid p(\theta) \geq \sigma\}$

3. Foreach θ in A do

3a. Create $M_\theta = \{(x, y) \mid \theta(x, y) = \theta\}$

3b. Dilate M_θ

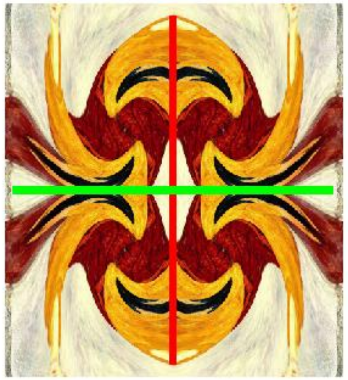
3c. Find the connected components R_θ^j

3d. Find the *Centroid* and *Major axis* of R_θ^j

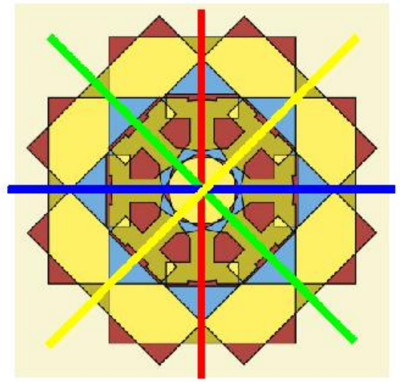
End

Multiple Reflection Symmetry Results

Number of symmetries detected for scale 1 : 2

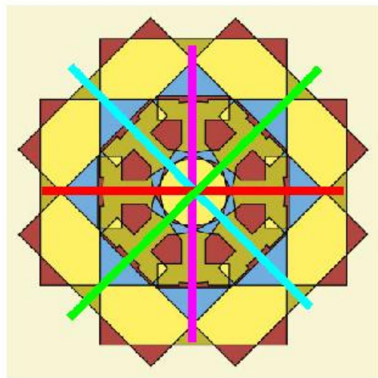


Number of symmetries detected for scale 1 : 4

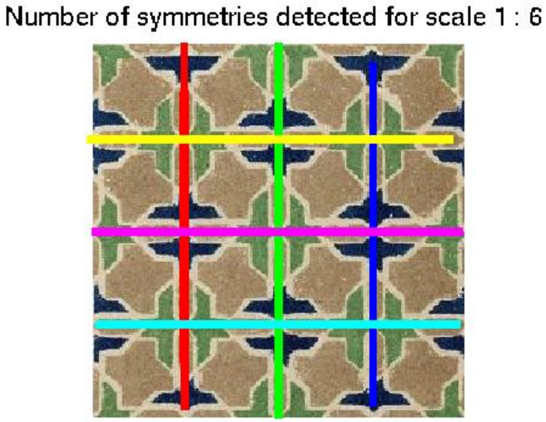


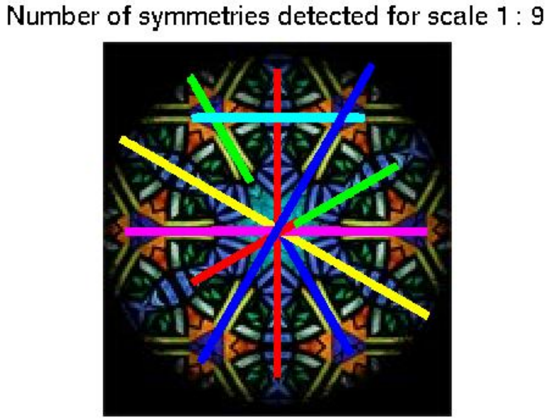
Number of symmetries detected for scale 2 : 2

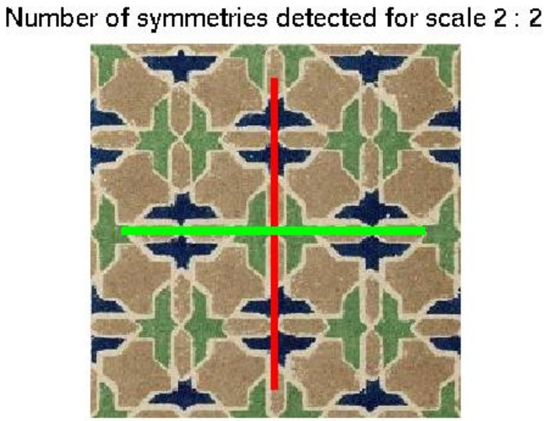
Number of symmetries detected for scale 2 : 4

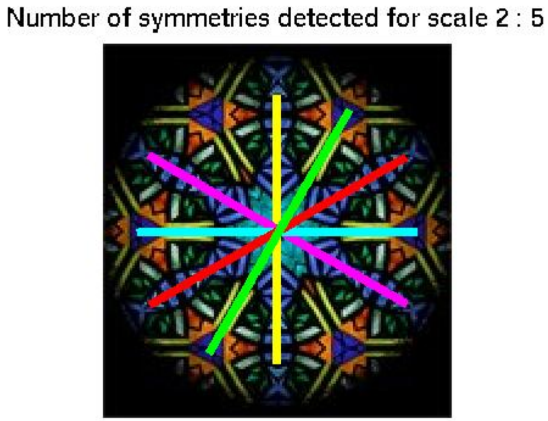


Multiple Reflection Symmetry Results

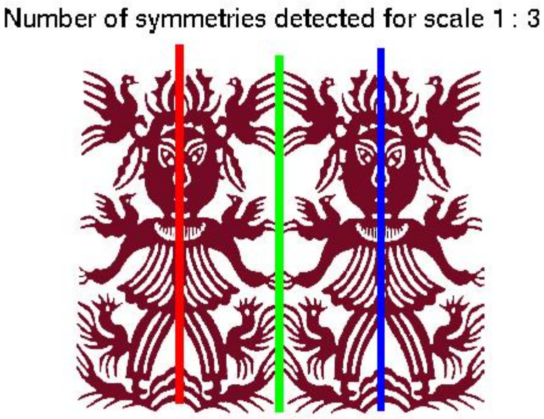


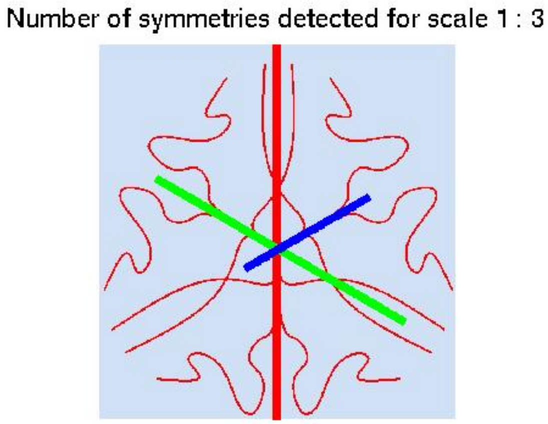


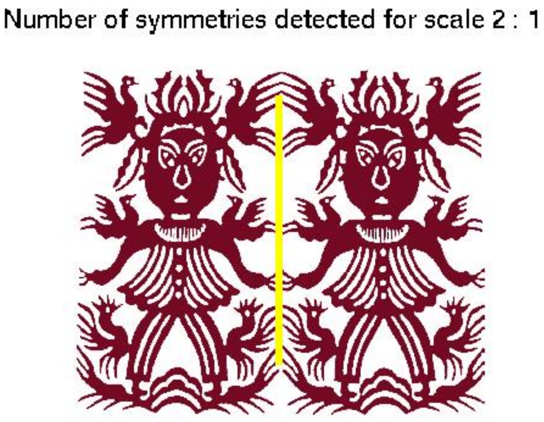


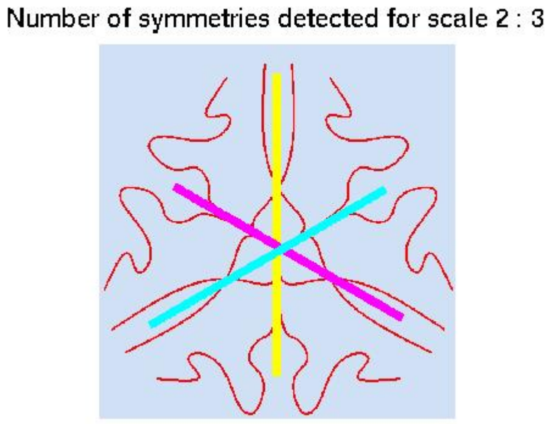


Multiple Reflection Symmetry Results









Detecting Multiple Rotation Symmetry

Algorithm

1. Let $p(\theta)$ be the distribution of angles θ (symmetry axis)

2. Initialize $G = \varnothing$

3. Create $A = \{\theta \mid p(\theta) \geq \sigma\}$

4. Foreach θ in A do

4a. Create $M_\theta = \{(x,y) \mid \theta(x,y) = \theta\}$

4b. Dilate M_θ

4c. G = G + M_θ

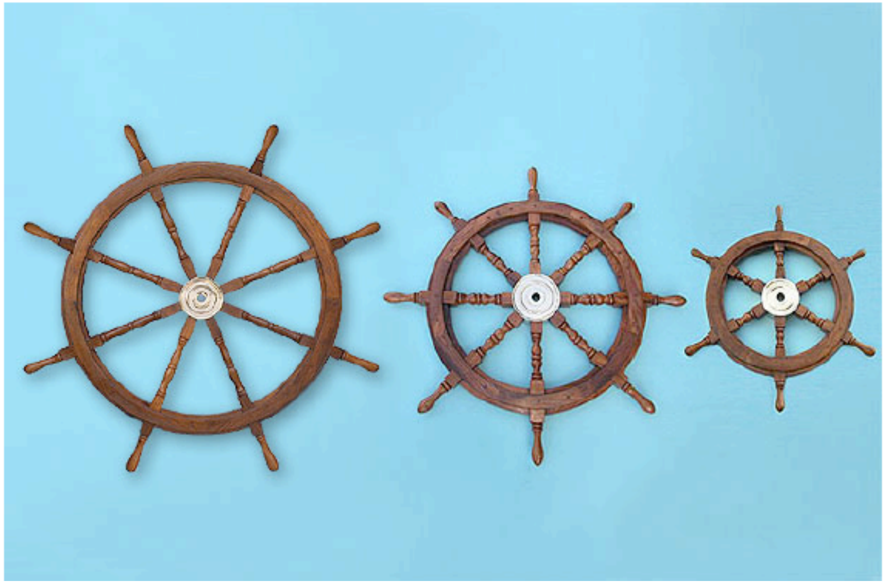
End

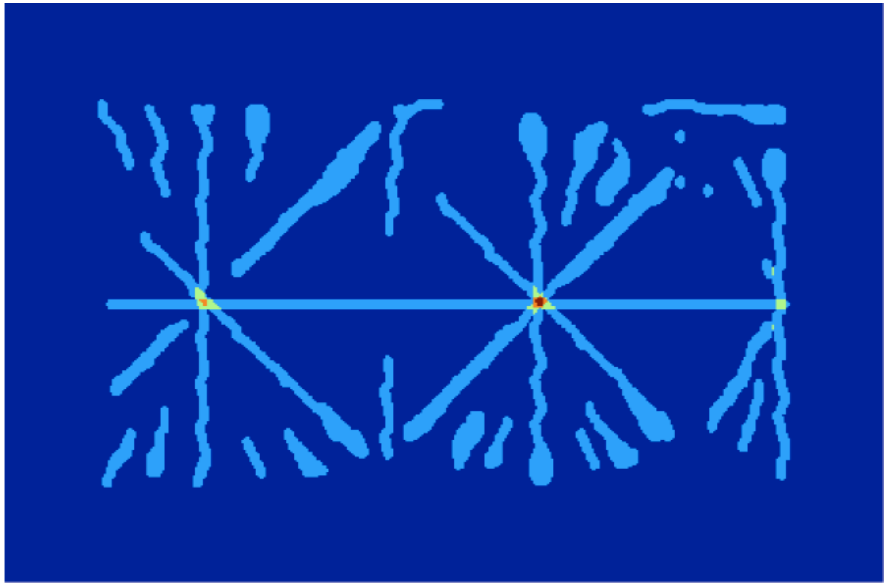
5. Threshold and dilate G

6. Find connected components R^j in G

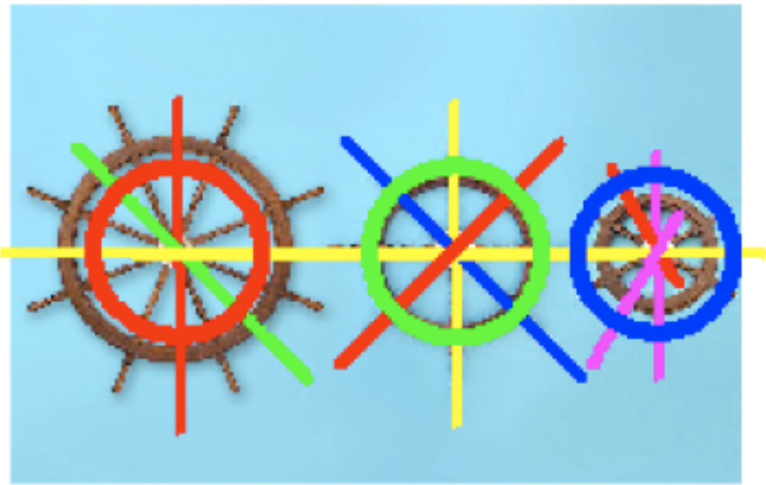
7. Find the *Centroid* and *Major axis* of R^j

Example

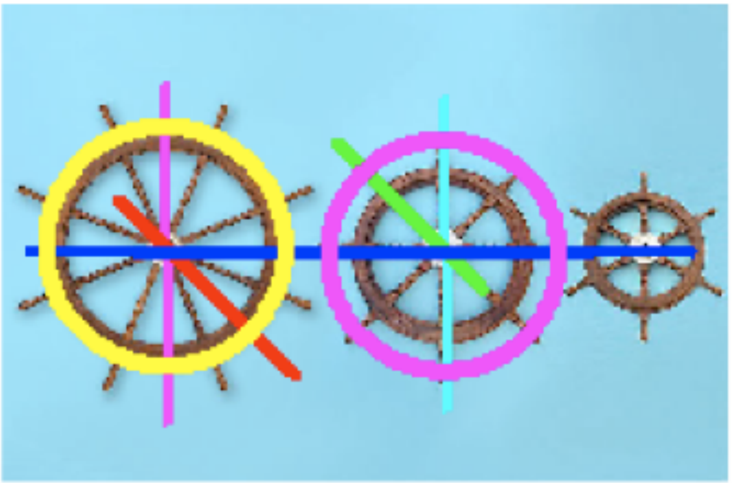




Number of Rotation symmetries for scale 1 : 3

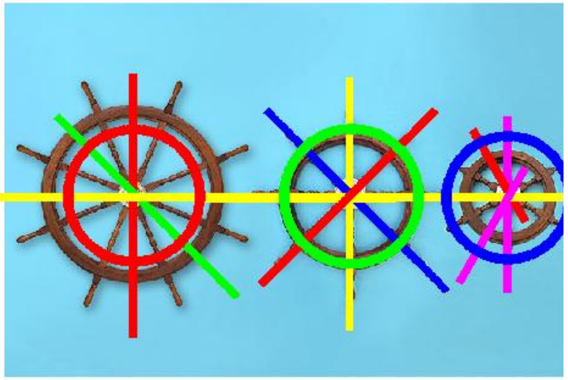


Number of Rotation symmetries for scale 2 : 2

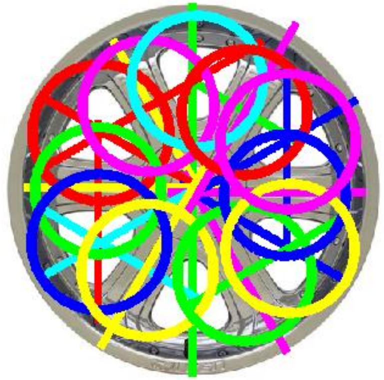


Multiple Rotation Symmetry Results

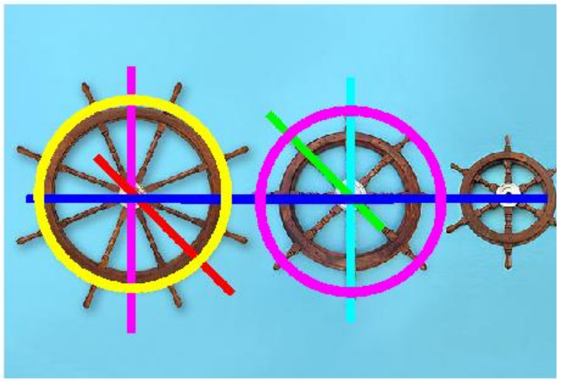
Number of Rotation symmetries for scale 1 : 3



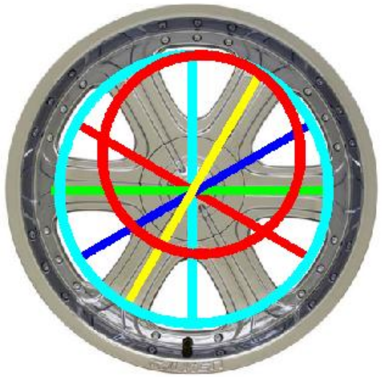
Number of Rotation symmetries for scale 1 : 11



Number of Rotation symmetries for scale 2 : 2



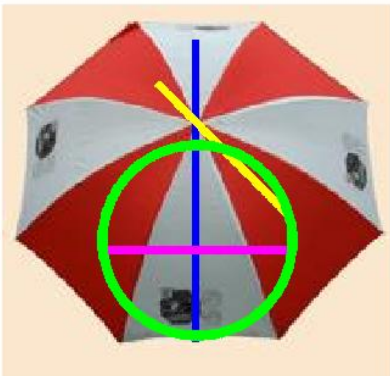
Number of Rotation symmetries for scale 2 : 2



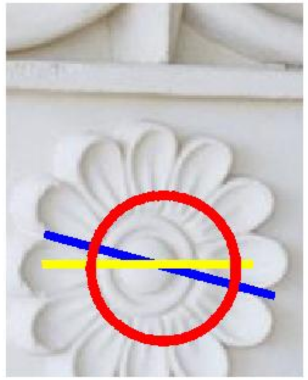
Multiple Rotation Symmetry Results

Number of Rotation symmetries for scale 1 : 1

Number of Rotation symmetries for scale 2 : 1



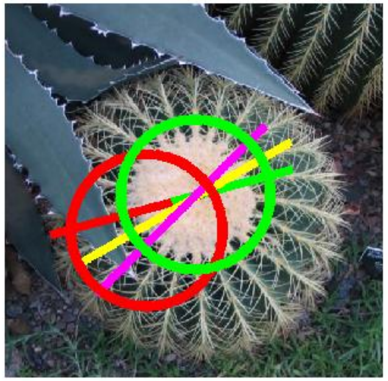
Number of Rotation symmetries for scale 1 : 1



Number of Rotation symmetries for scale 2 : 0

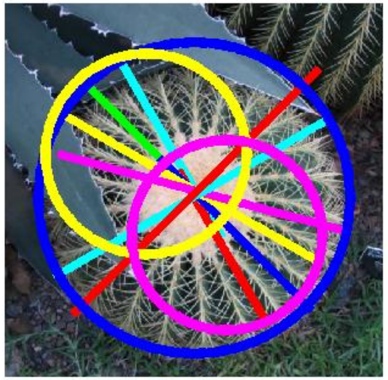
Multiple Rotation Symmetry Results

Number of Rotation symmetries for scale 1 : 2



Number of Rotation symmetries for scale 1 : 1

Number of Rotation symmetries for scale 2 : 3

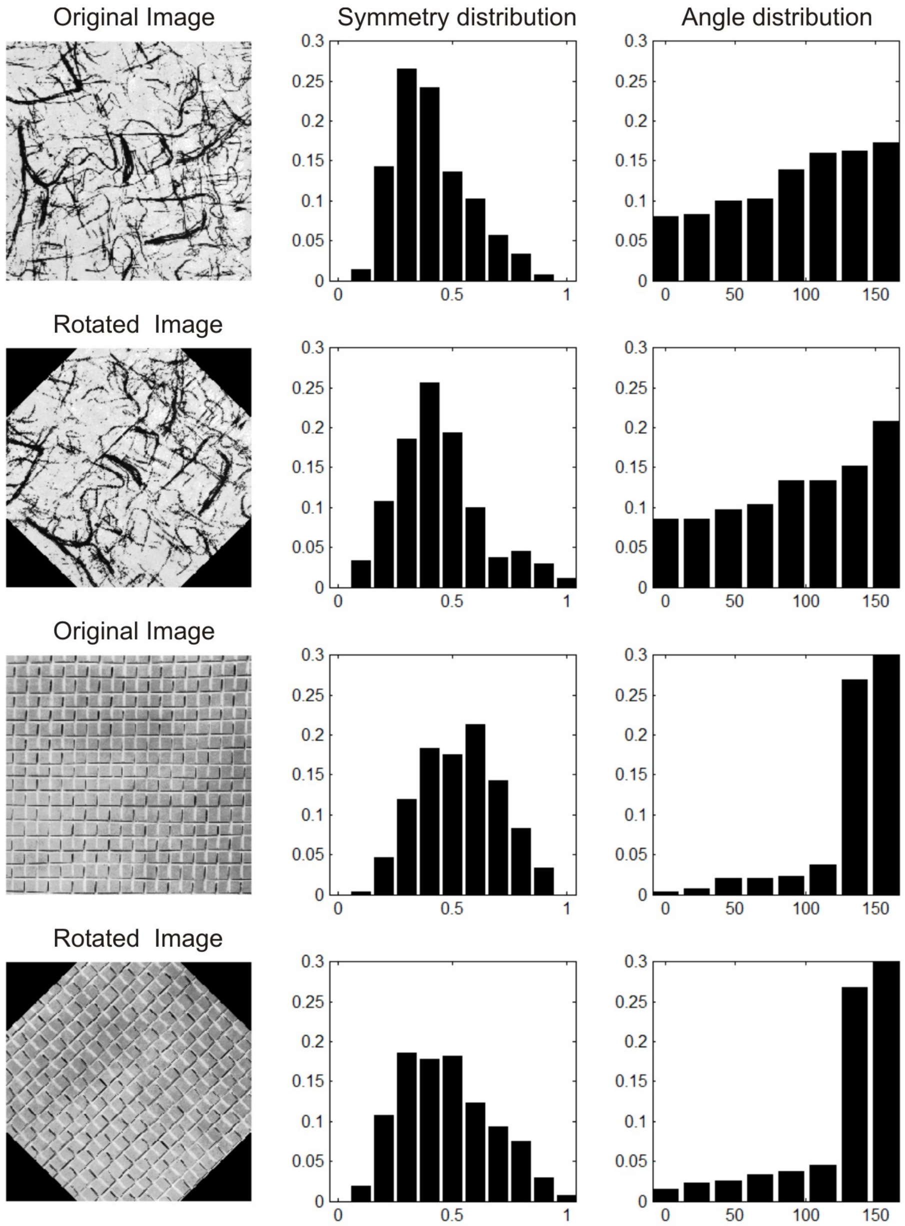


Number of Rotation symmetries for scale 2 : 0

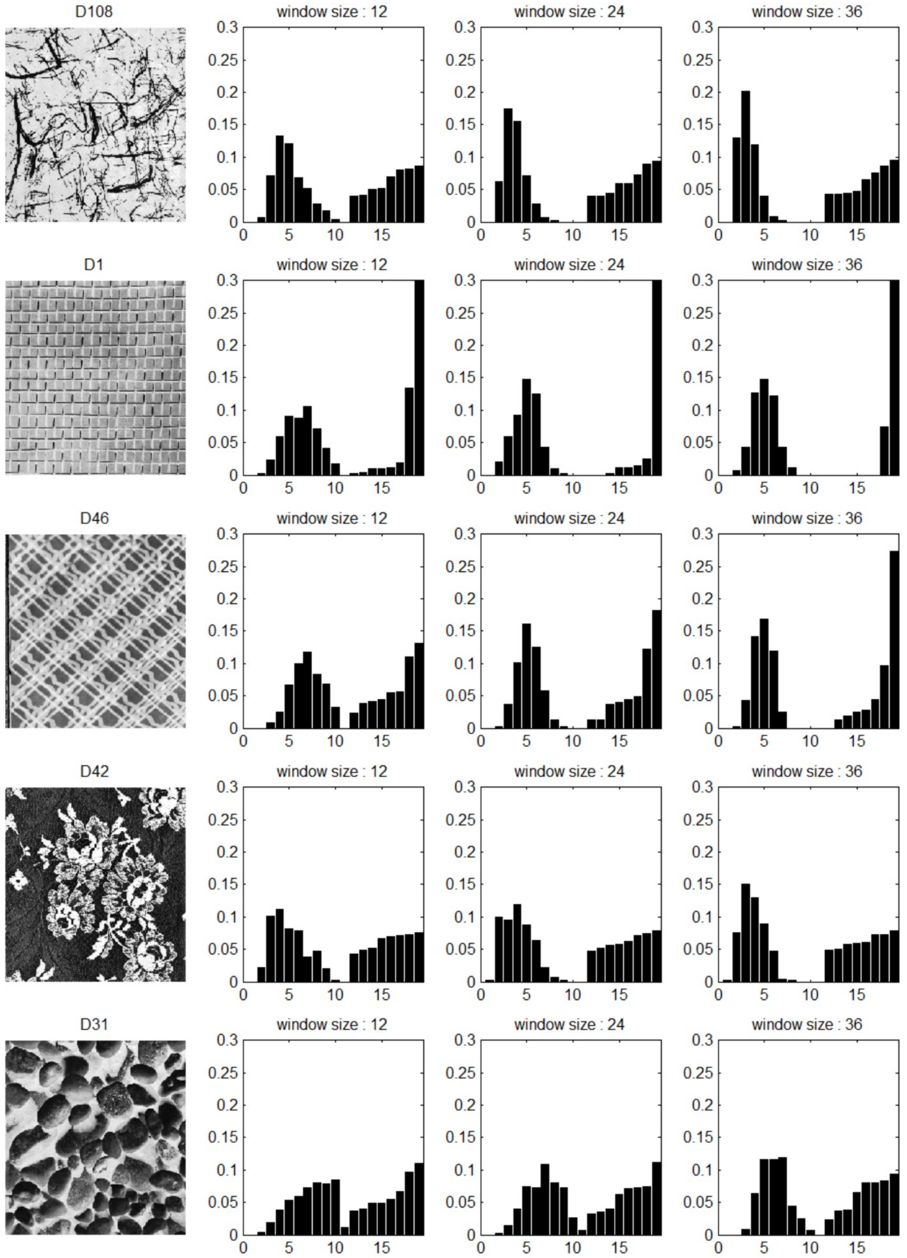
Ongoing work

- Find interest points
- Determine symmetry axis
- Classification using the distribution of the local symmetries
- Image Registration/Matching

Texture separation



Texture Separation



Symmetry Features and Classification

- The feature vector is based on

   1. Distribution of symmetry with 11 histogram bins in the range [0,1], with bin width 0.1.
   2. Sorted distribution of symmetry directions (14 different directions are used)
   3. Distribution of entropy with 5 bins in the range [0, 0.6], with bin width 0.15

- The histogram is then classified using SVMs.

Texture Separation

- Symmetry can be used to help the classification results between uniform and non-uniform textures.

Feature	Recognition rate (%) on subset of Brodatz dataset
Symmetry	72.98 ± 1.8
Textons [8]	95.97 ± 0.72
Textons + Symmetry (weight = 0.4)	98.27 ± 1.4

- The combination of textons and symmetry thus improves the result.

Texture Separation

- The texture datasets are UIUCTex, KTH-TIPS, Brodatz, and CUReT.

Database	UIUCTex	KTH-TIPS	Brodatz	CUReT
Ours	96.9 ±0.8	**98.1 ±1.1**	94.0 ±0.9	98.5 ±0.2
Kondra [13]	92.9 ±1.2	97.7 ±0.8	92.3 ±1.0	97.0 ±0.4
Zhang [30]	**98.3 ±0.5**	95.5 ±1.3	**95.4 ±0.3**	95.3 ±0.4
Hayman [7]	92.0 ±1.3	94.8 ±1.2	95.0 ±0.8	**98.6 ±0.2**
VZ-joint [28]	78.4 ±0.9	92.4 ±1.4	92.9 ±1.0	96.0 ±0.7
Lazebnik [14]	96.4 ±2.0	91.3 ±2.1	89.8 ±0.8	72.5 ±0.4
G. Gabor	65.2 ±2.0	90.0 ±2.0	87.9 ±1.0	92.4 ±0.5

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