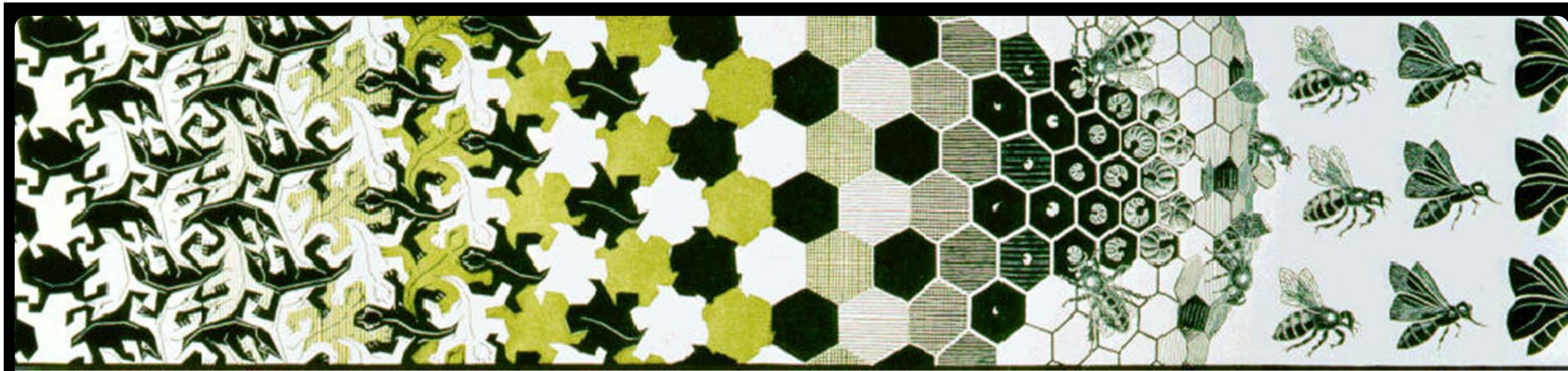


Metamorphosis III 1967-1968, *M.C. Escher*

Symmetry-Growing for Detecting Skewed Rotational Symmetry



Hyojin Kim, Minsu Cho, and Kyoung Mu Lee

Department of EECS

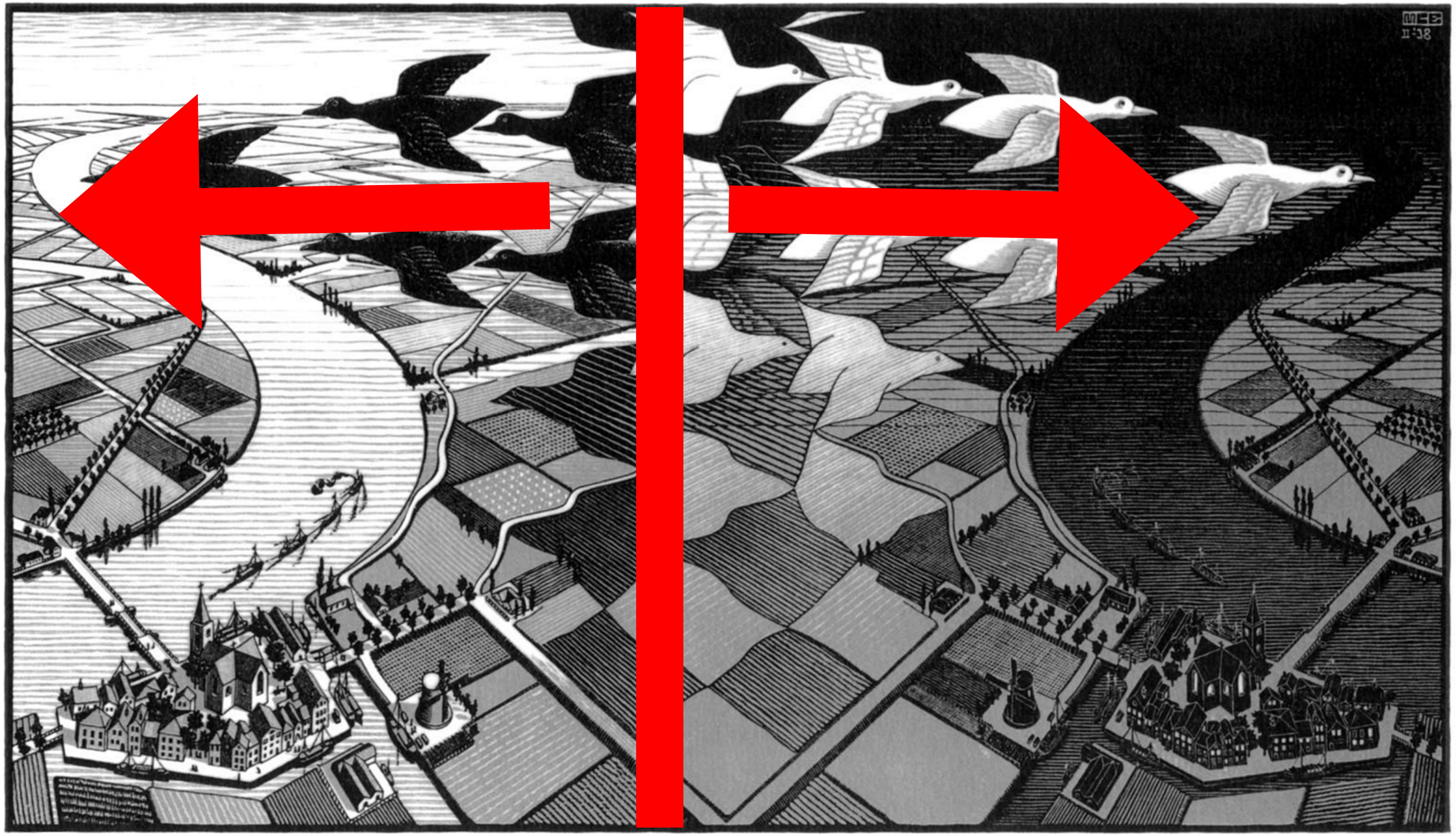
Seoul National University, Korea

ComputerVisionLab
Seoul National University

CVPR 11 Workshop :Symmetry Detection from Real World Images

20 COLORADO 11

Day and Night



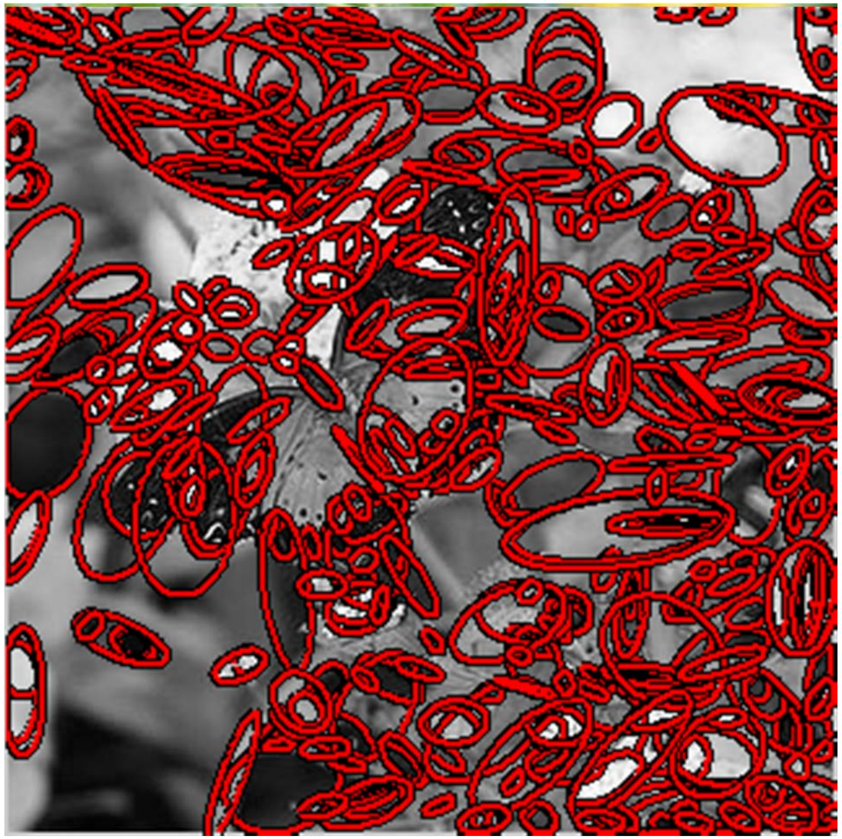
Day and Night, 1938, *M.C. Escher*

Symmetry-Growing

A given image

M.Cho and K.M.Lee, *Bilateral Symmetry Detection via Symmetry-Growing*, BMVC 2009

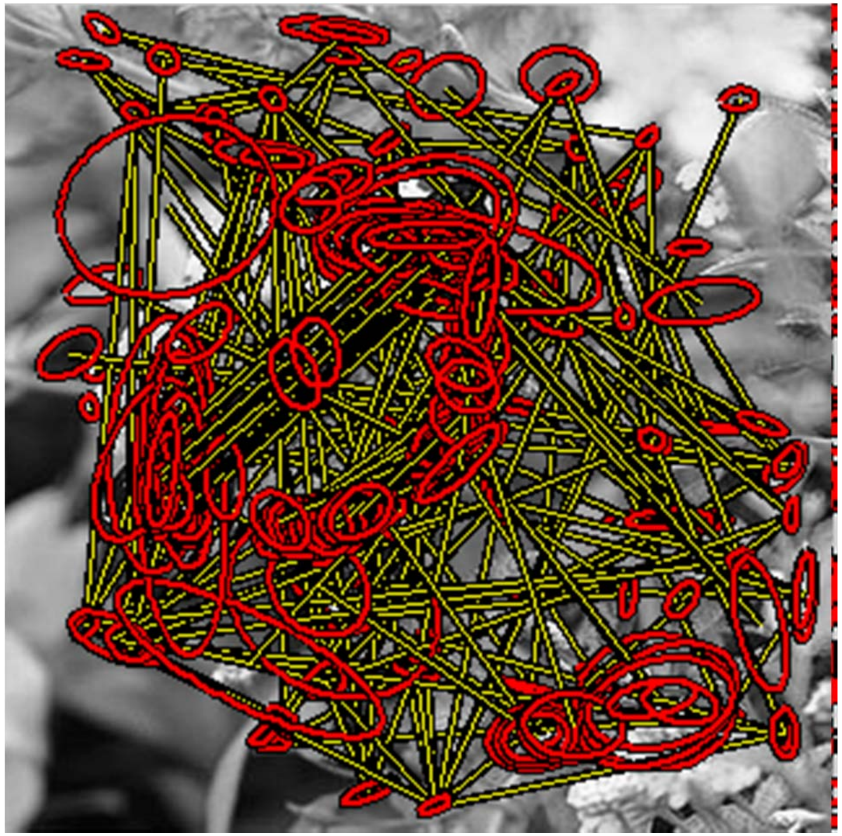
Symmetry-Growing



Local feature detection

M.Cho and K.M.Lee, *Bilateral Symmetry Detection via Symmetry-Growing*, BMVC 2009

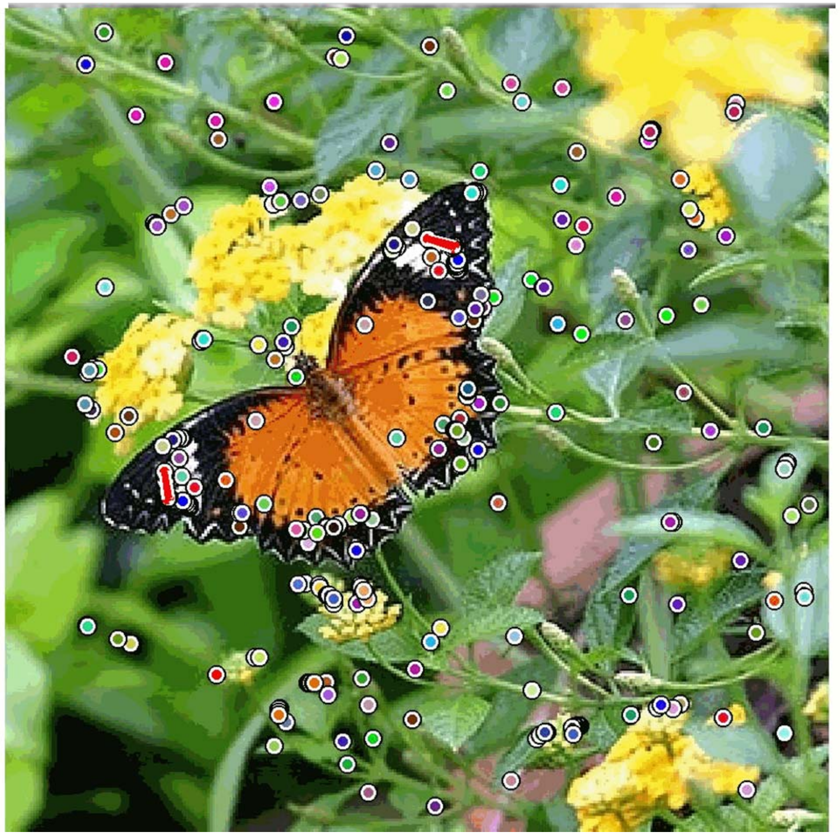
Symmetry-Growing



Symmetry seed extraction

M.Cho and K.M.Lee, *Bilateral Symmetry Detection via Symmetry-Growing*, BMVC 2009

Symmetry-Growing



Symmetry-growing

M.Cho and K.M.Lee, *Bilateral Symmetry Detection via Symmetry-Growing*, BMVC 2009

Symmetry-Growing

Symmetry Verification

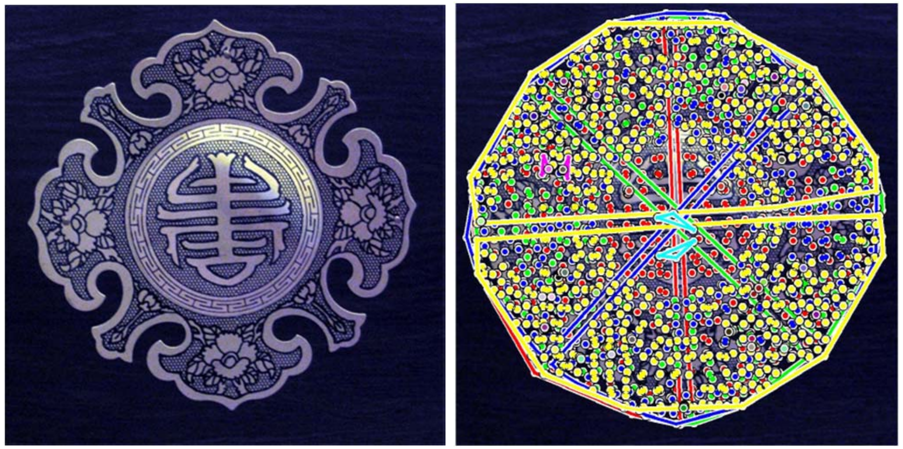
M.Cho and K.M.Lee, *Bilateral Symmetry Detection via Symmetry-Growing*, BMVC 2009

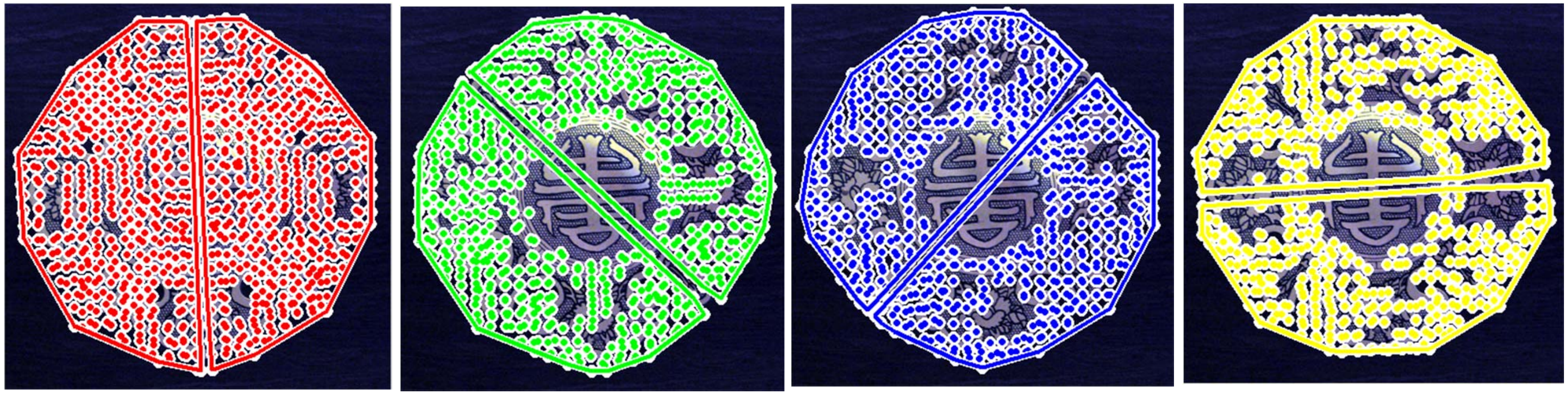
Symmetry-Growing

- Our multi-layer growing framework enables overlapping symmetries & robust feature grouping

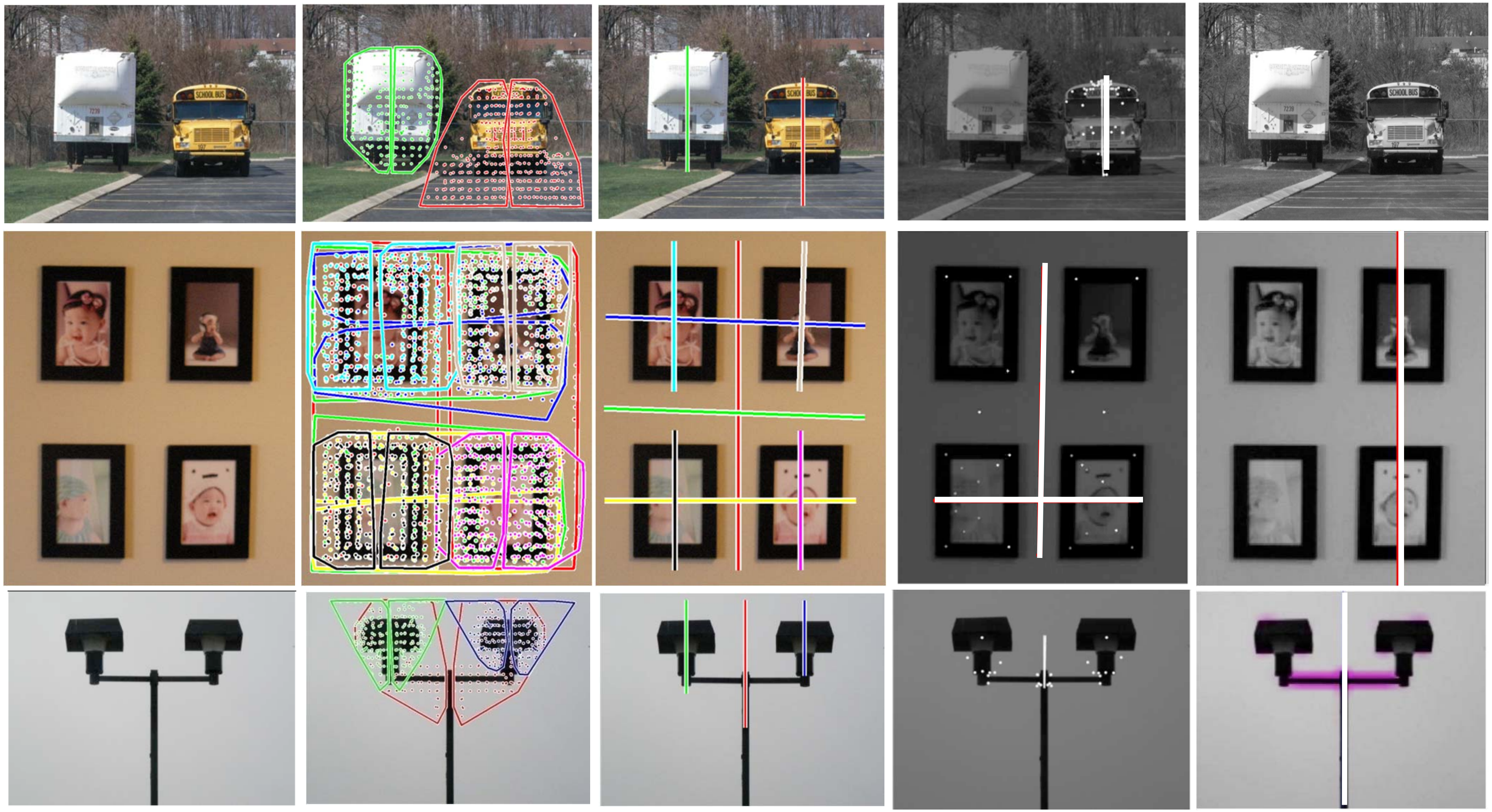
Symmetry-Growing

- Locally symmetric parts are inferred by the feature distribution



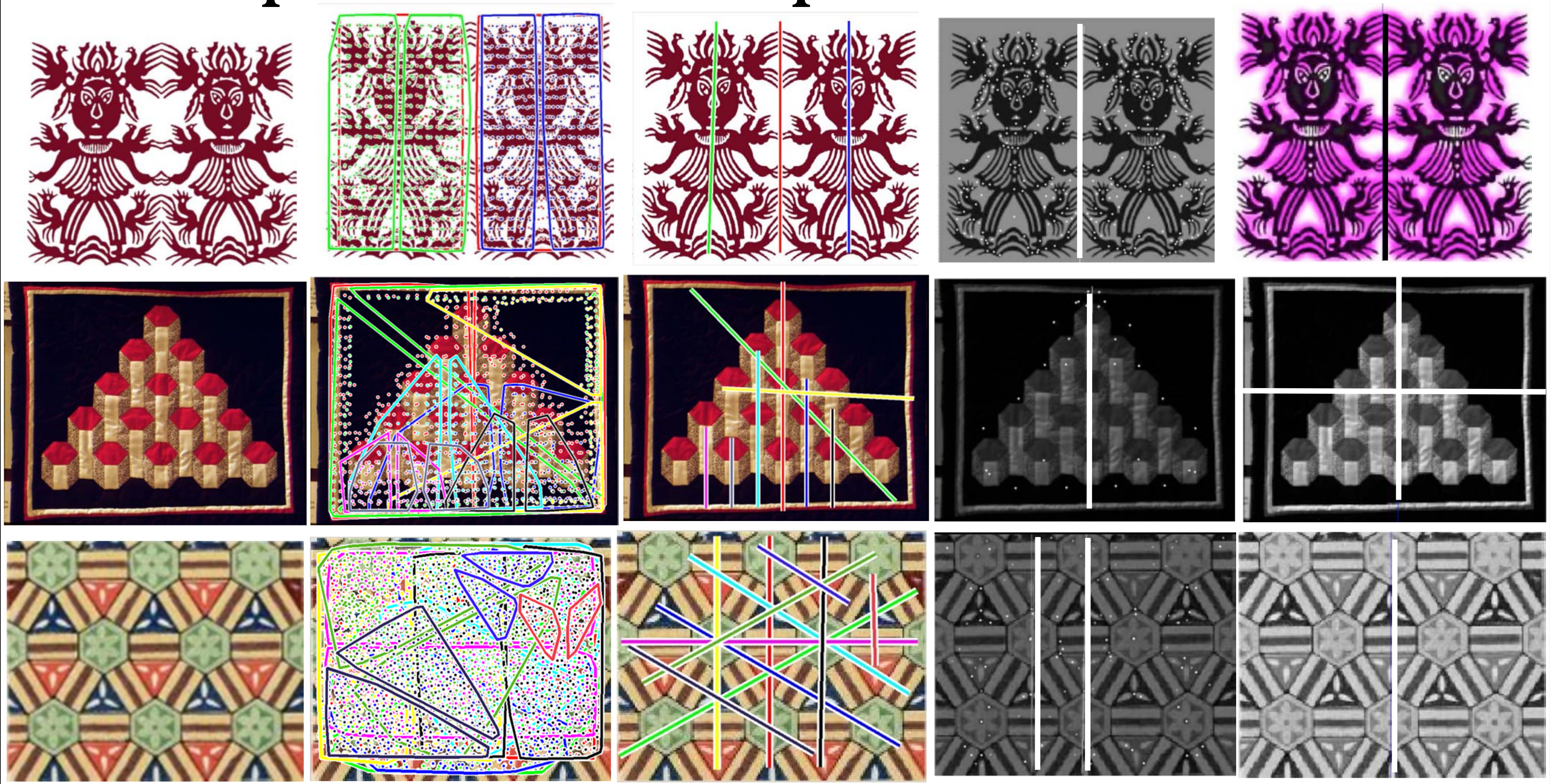


Comparative examples



Input **Our result** **LE06** **LHS05**

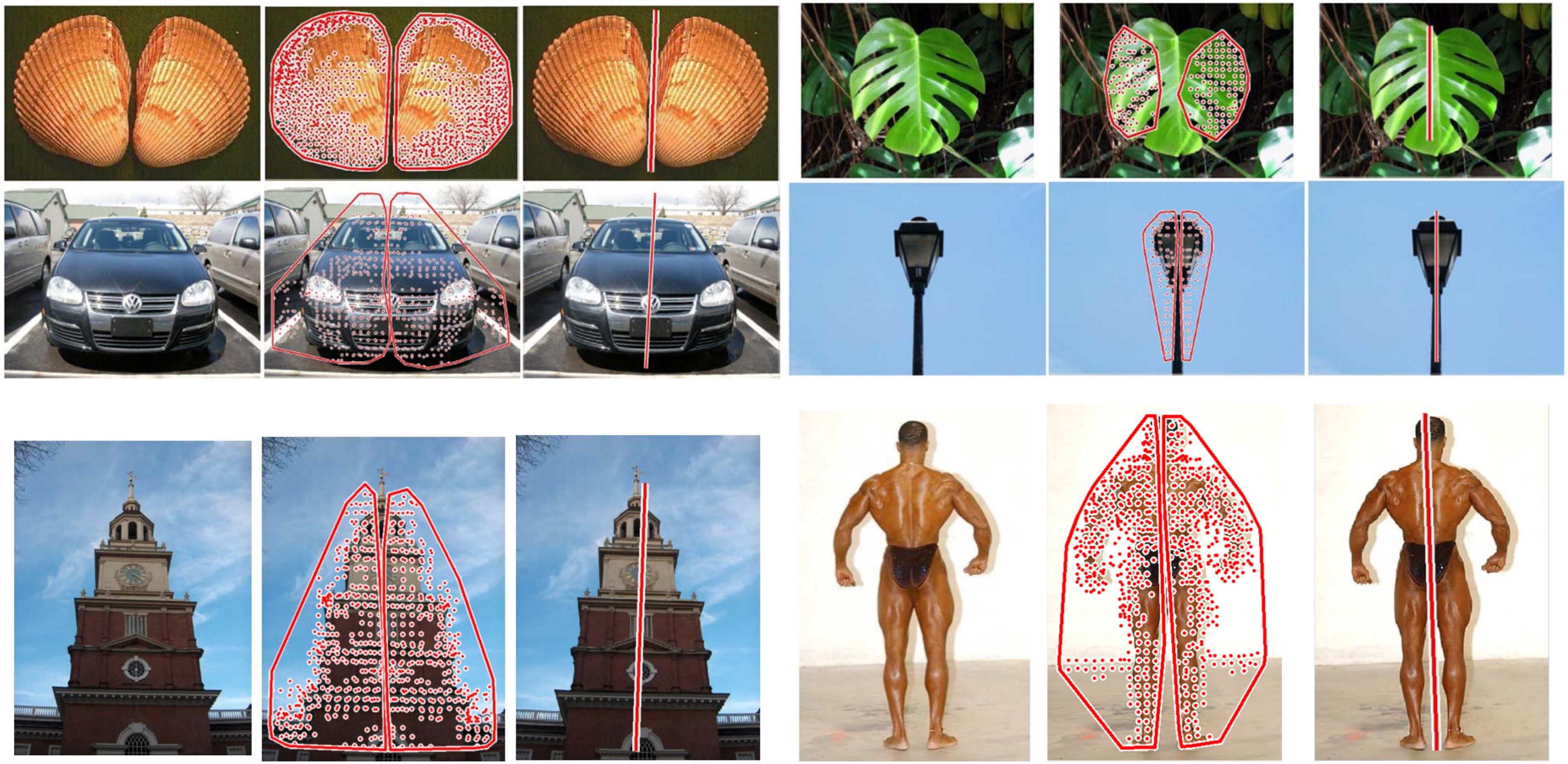
Comparative examples



Input **Our result** **LE06** **LHS05**

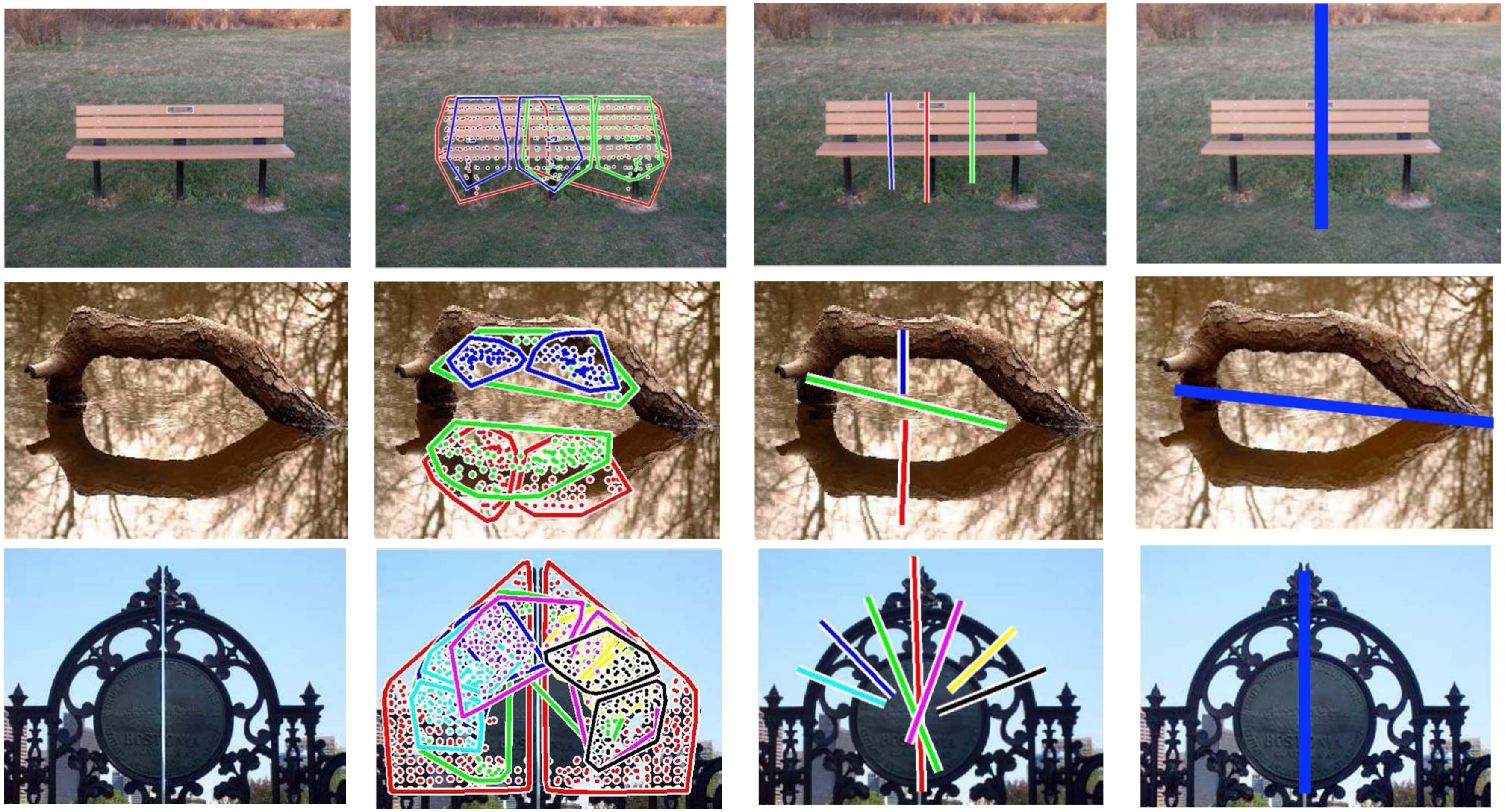
Experimental Results

- Our results on images with single symmetry patterns



Detected Symmetries beyond Ground Truth

- Examples with a single symmetry pattern



Input **Our result** **Ground Truth**

Experimental Results

- Quantitative results
  - Measure: sensitivity & false positive rate

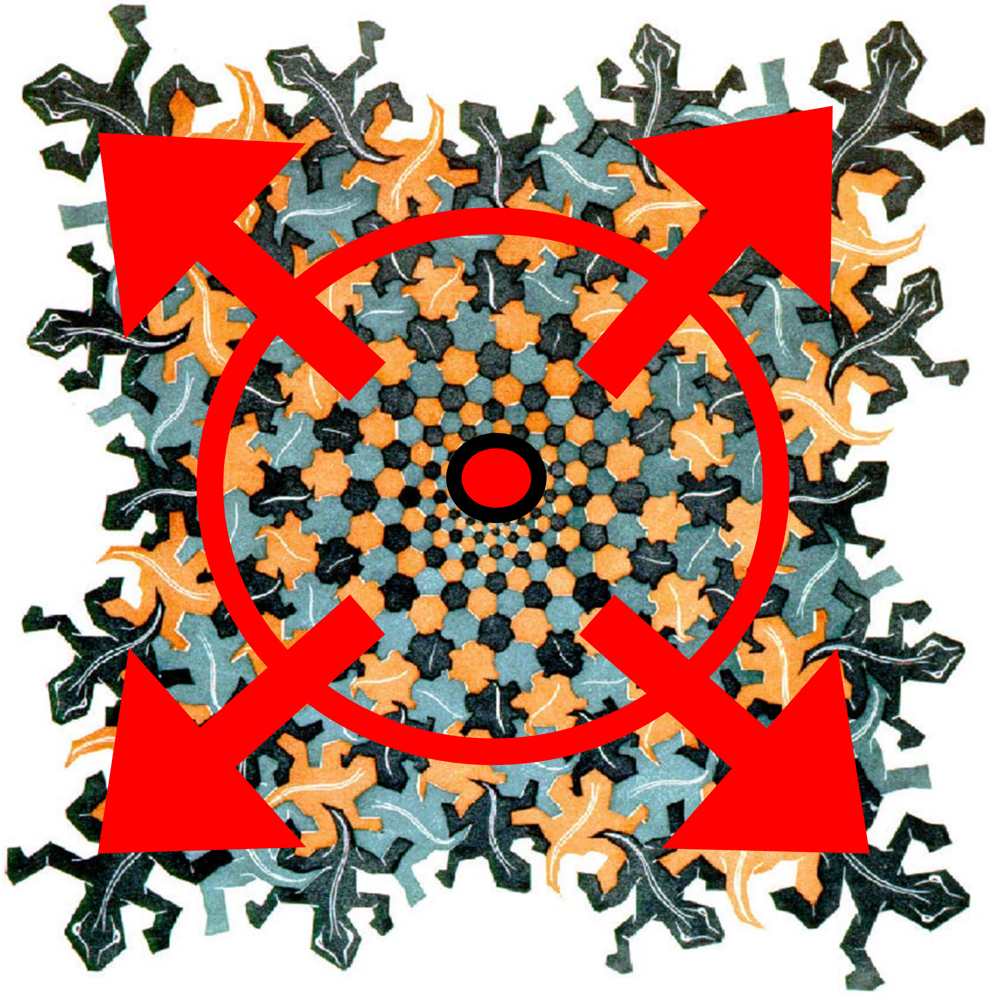
$$S_0 = TP/GT \quad R_{FP} = FP/GT$$

  - On all the 83 images of PSU Ref. symmetry dataset
    - Ground truth & other results from M. Park *et al.'s CVPR2008*

Image Type	Synthetic Single			Synthetic Multiple		
Algorithm	LE06	LHS05	**Ours**	LE06	LHS05	**Ours**
S_0	92%	62%	**100%**	35%	28%	**77%**
R_{FP}	15%	0%	**15%**	4%	8%	**33%**
Image Type	Real Single			Real Multiple		
Algorithm	LE06	LHS05	**Ours**	LE06	LHS05	**Ours**
S_0	84%	29%	**94%**	43%	18%	**68%**
R_{FP}	68%	3%	**69%**	44%	0%	**17%**

- Overall S_0: 84% (+20% than LE06), R_{FP}: 38% (-4% than LE06)

Development II

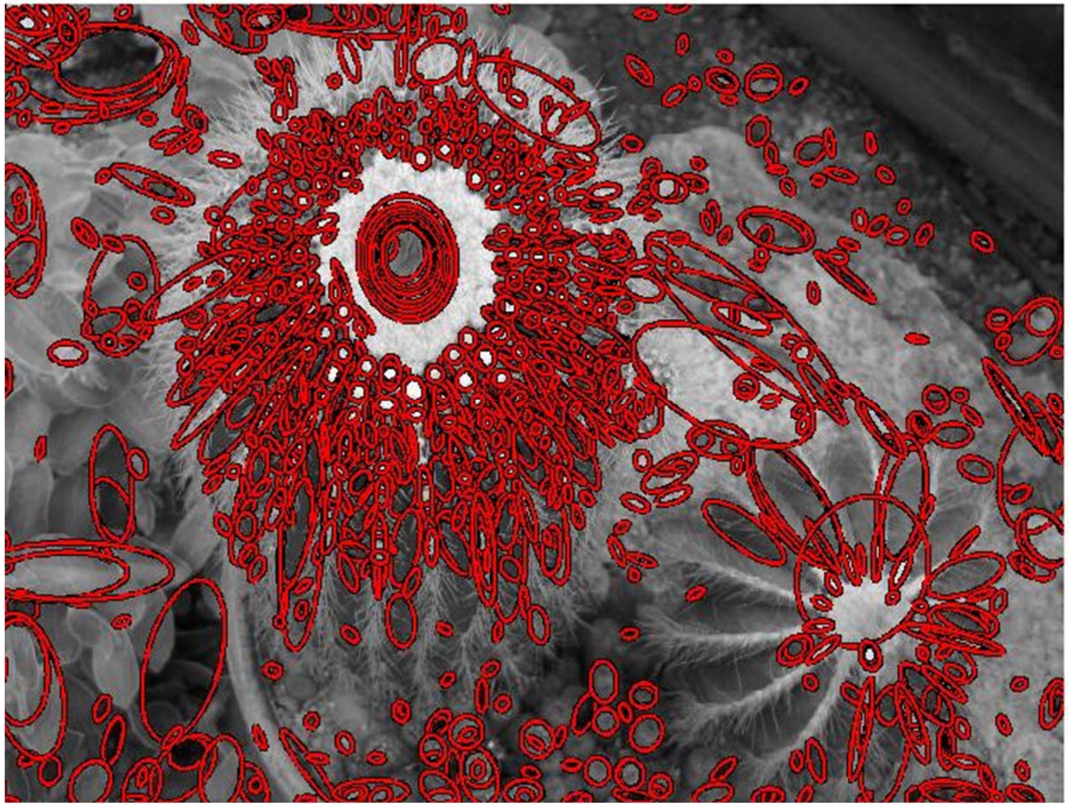


Development II 1939, *M.C. Escher*

Symmetry-Growing

A given image

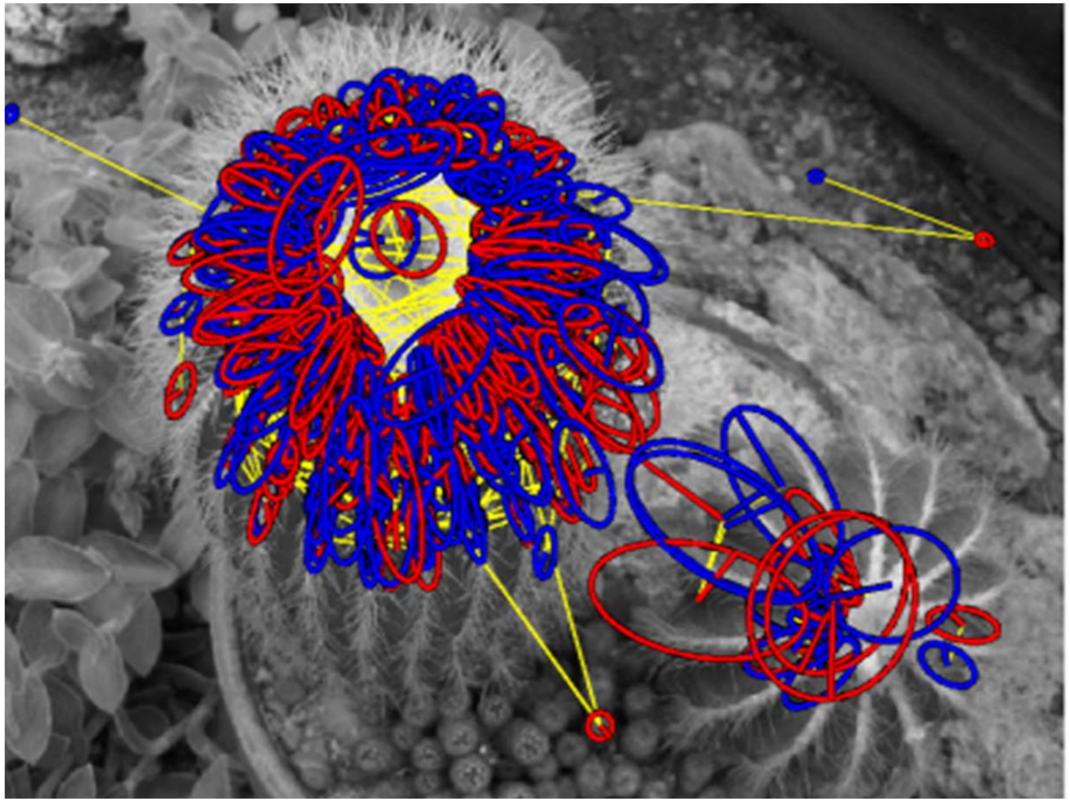
Symmetry-Growing



Local feature detection

Computer**Vision**Lab
Seoul National University

Symmetry-Growing

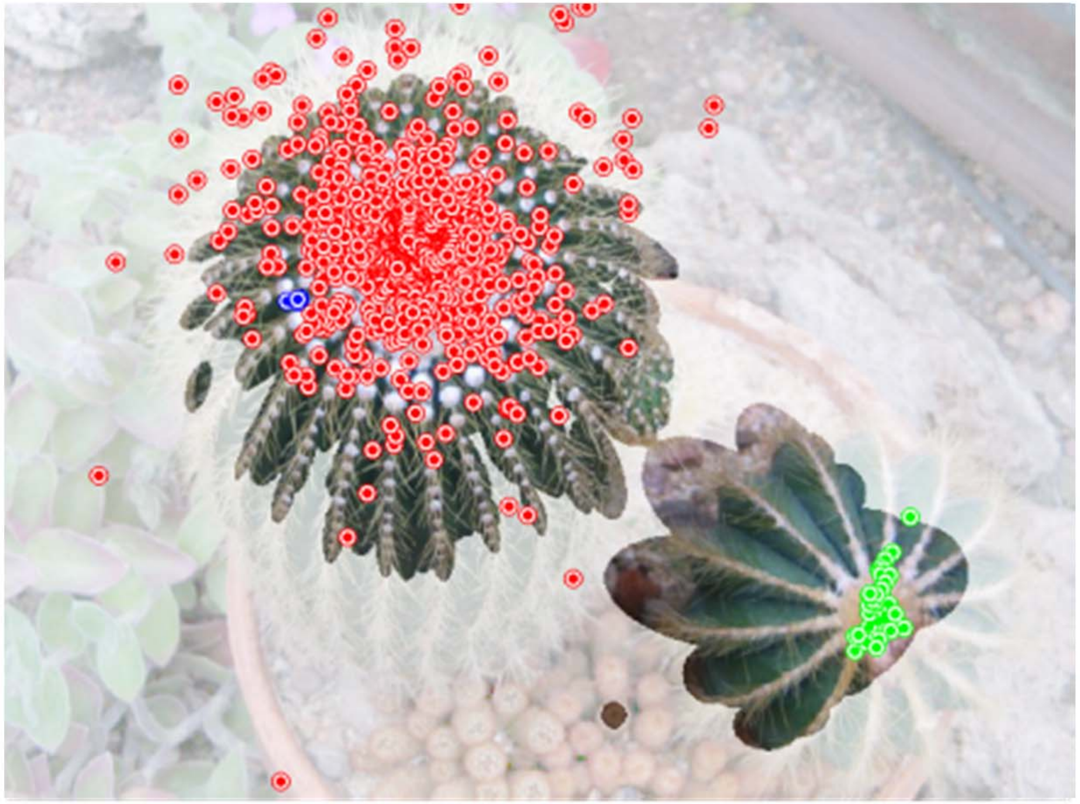


Symmetry seed extraction

Symmetry-Growing

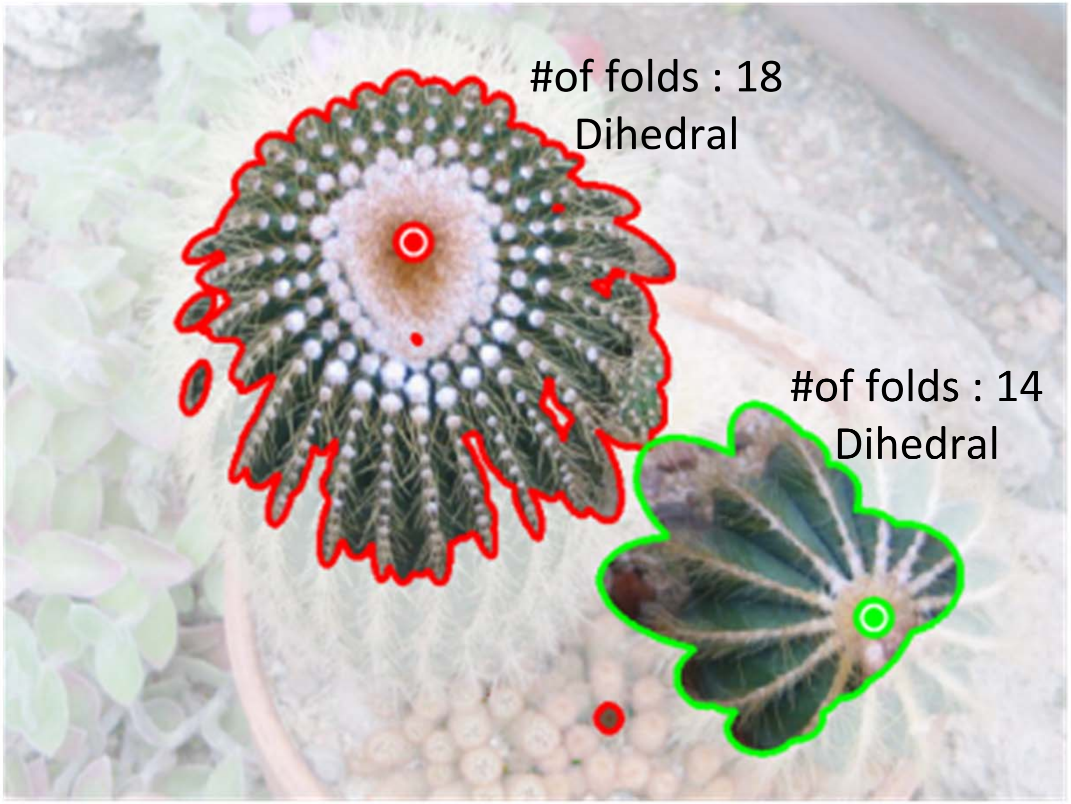
Symmetry-growing

Symmetry-Growing



Symmetry Analysis

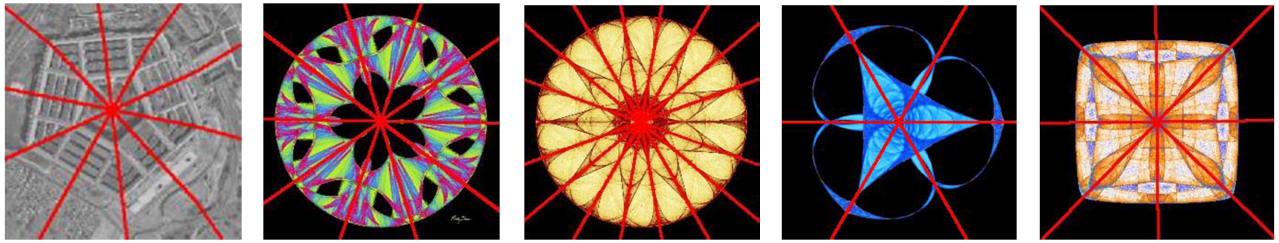
Symmetry-Growing



Symmetry Analysis

Previous Approach

- Global methods: the entire image as a signal



Kellerand.Shkolnisky *IEEE Tran.Image Proc. 2006*

  - Not robust to background clutter

- Local methods: Grouping symmetric sets of local features

G. Loy and J.O. Eklundh *ECCV2006*

  - efficiently detect local symmetries against background clutters
  - But, largely influenced by initial feature detection step

Our Contribution

- Robust detection method via symmetry-growing

Global methods
Exploit global information, but vulnerable to clutter

Local methods
Robust to clutter, but restricted to detected features

Our method
Explore image regions to find further symmetry beyond detected symmetric features

skew-robust axis estimation
adapt to affine deformation when analyzing symmetry from affine covariant feature pairs

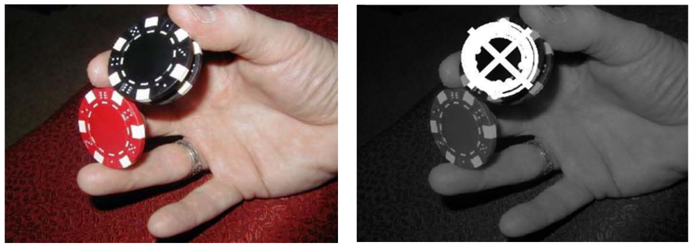
Our Contribution

- Rotational symmetry detection robust to

Background Clutters

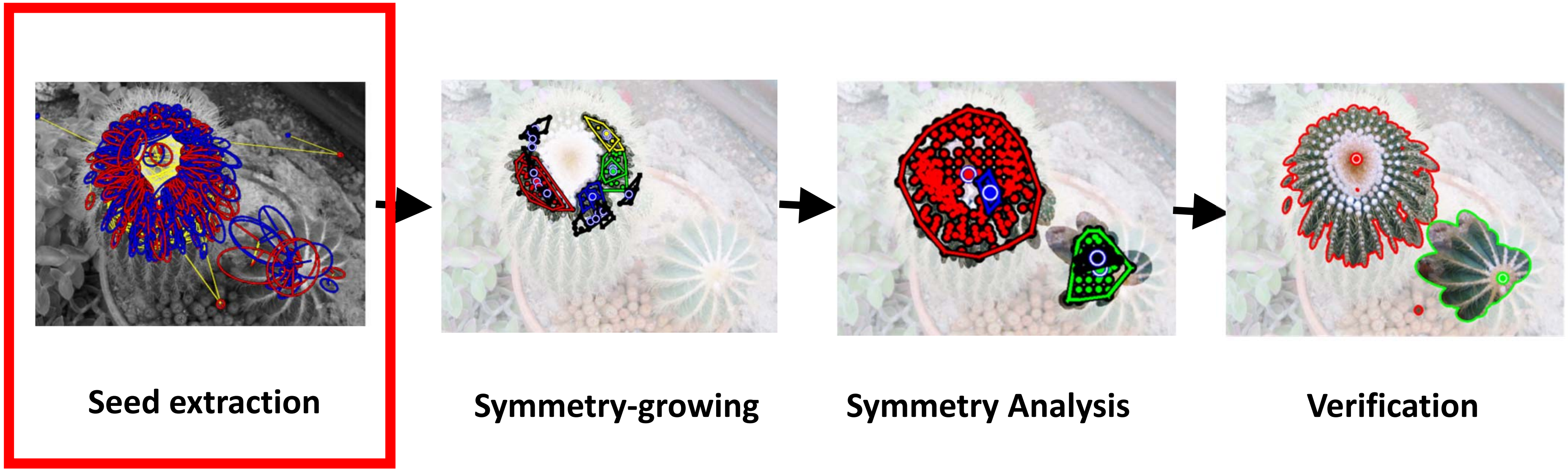
Insufficient Local Features

Distortion of Symmetry



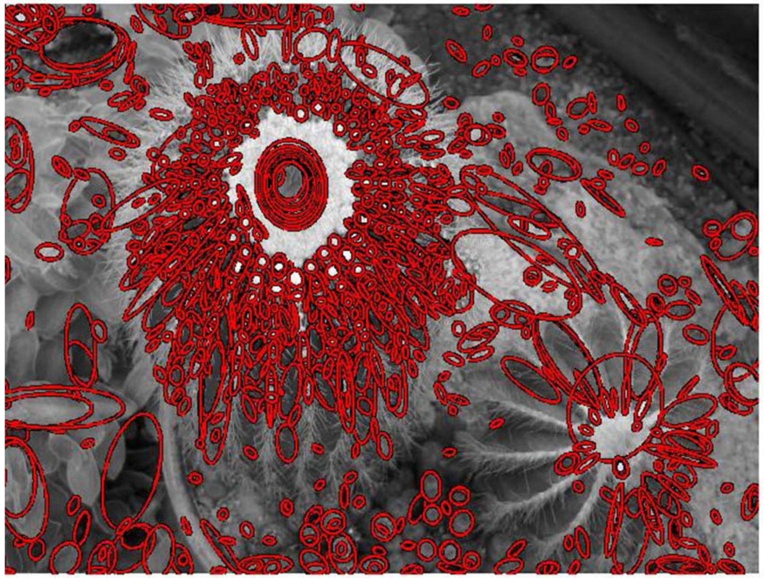
Step#1: Seed Extraction

Goal: Extract seed matches for symmetric patterns from the given image



Local feature detection

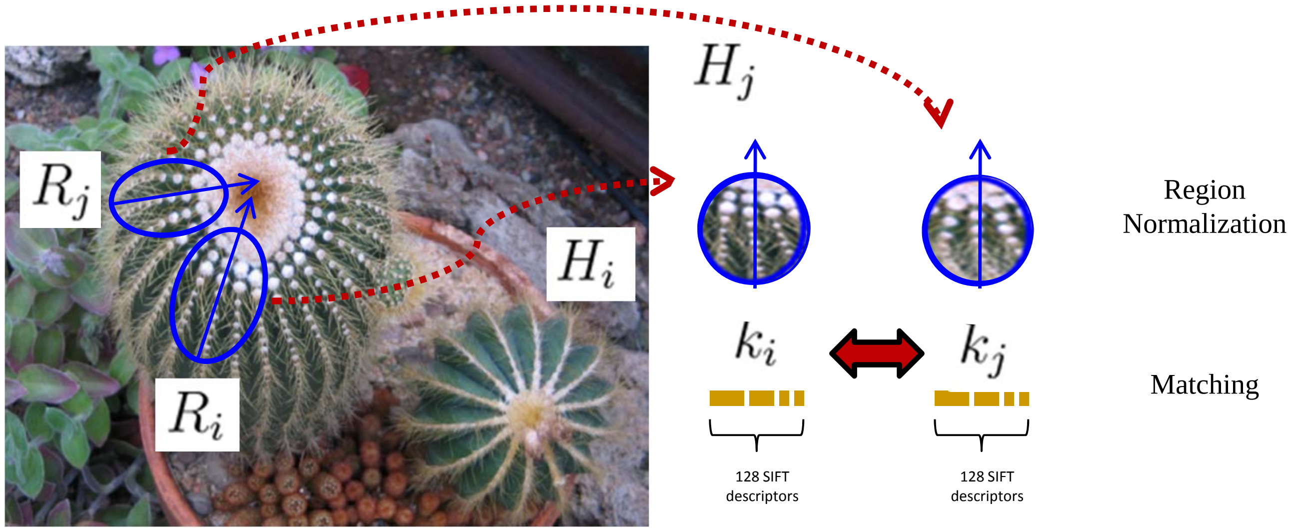
- Any of affine-covariant feature detectors



- MSER (Matas *et al* '02)
- Harris-affine (Mikolajczyk and Schmid '04)
- Edge-laplace (Mikolajczyk and Schmid '04)
- …

Symmetric feature pairs

- Mirror matching with normalized feature regions



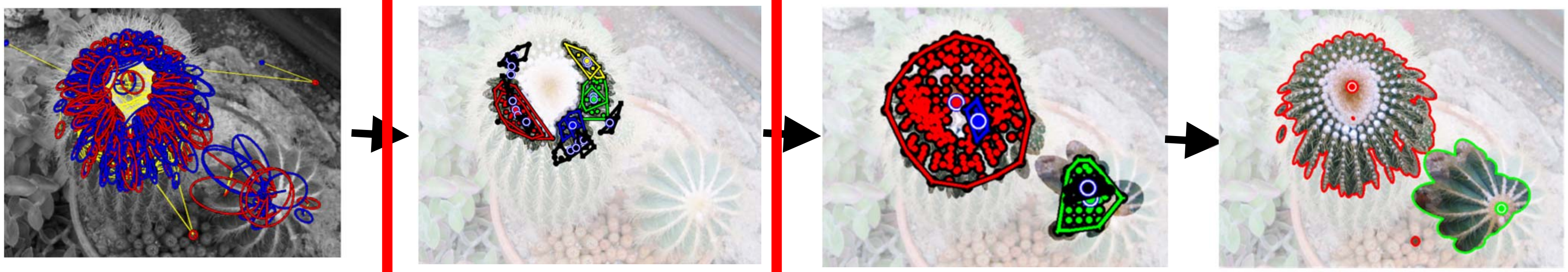
Region Normalizing Matrices:

$$H_i = R_{\angle o_i}{}^{-1} \Sigma_i^{\frac{1}{2}}$$

$$H_j = R_{\angle o_j}{}^{-1} \Sigma_j^{\frac{1}{2}}$$

Step#2: Symmetry-Growing

Goal: Grow the obtained symmetry seeds by multi-layer symmetry-growing



Seed extraction

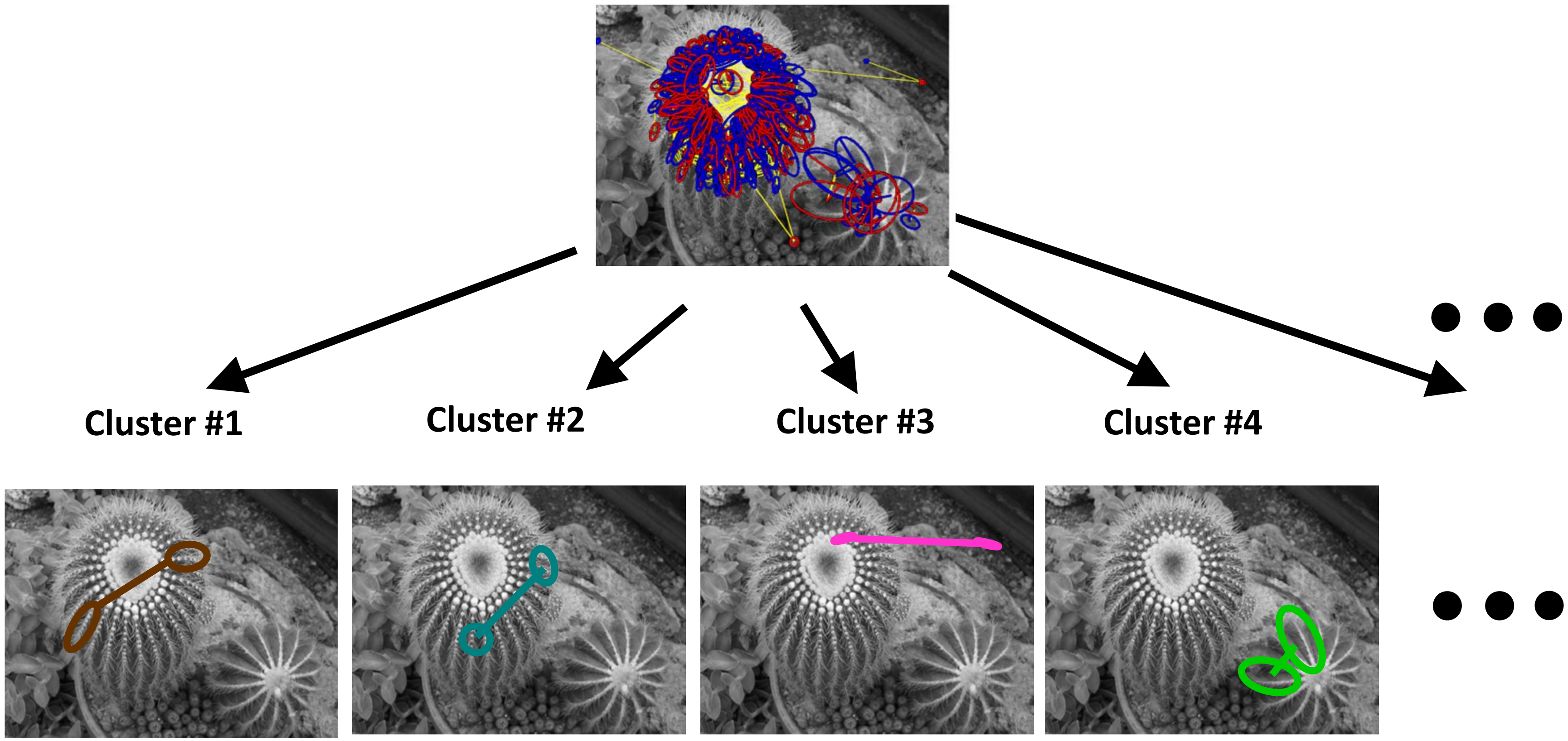
Symmetry-growing

Symmetry Analysis

Verification

Symmetry Cluster Initialization

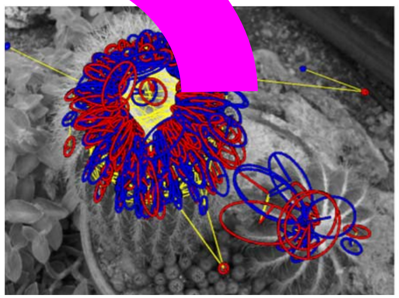
- Initially, each seed constitutes a singleton cluster



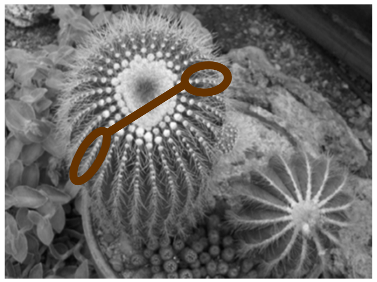
Supporter List Initialization

- Initialize *supporter list* as the set of seed matches

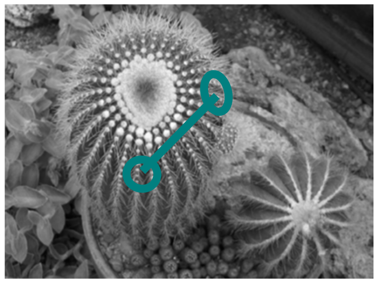
Supporter match list



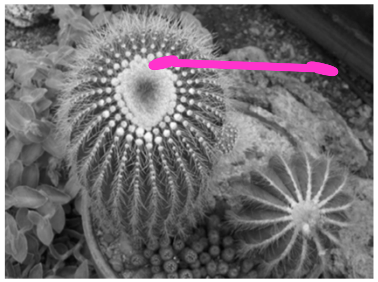
Cluster #1



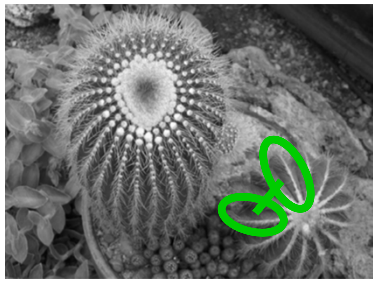
Cluster #2



Cluster #3



Cluster #4



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Iterative Growing Process

- Pick out the best supporter, and expand its cluster

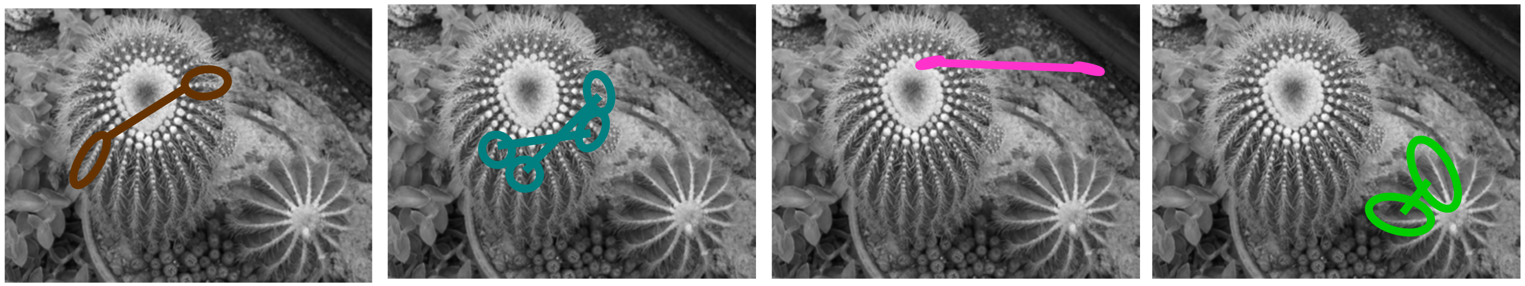
Supporter match list

The best supporter

Add expanded matches

Expand its cluster

Cluster #1 **Cluster #2** **Cluster #3** **Cluster #4**



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Iterative Growing Process

- Pick out the best supporter, and expand its cluster

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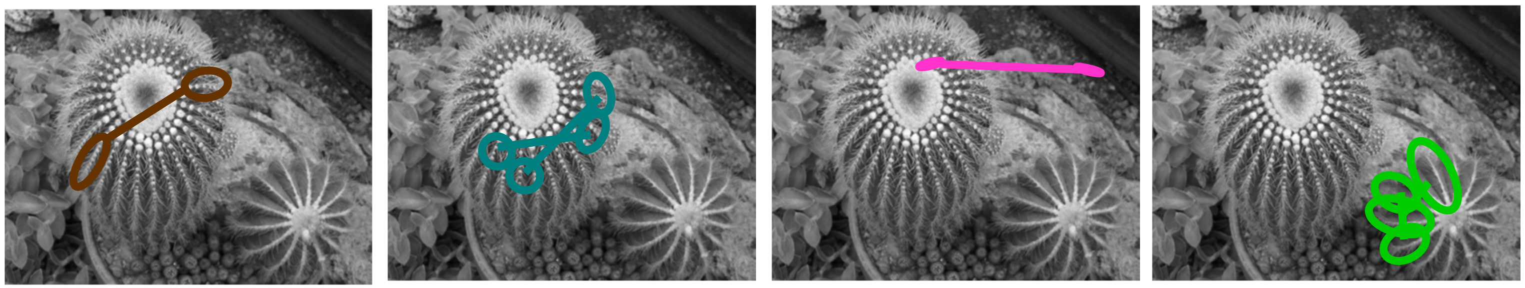
Cluster #1 **Cluster #2** **Cluster #3** **Cluster #4**



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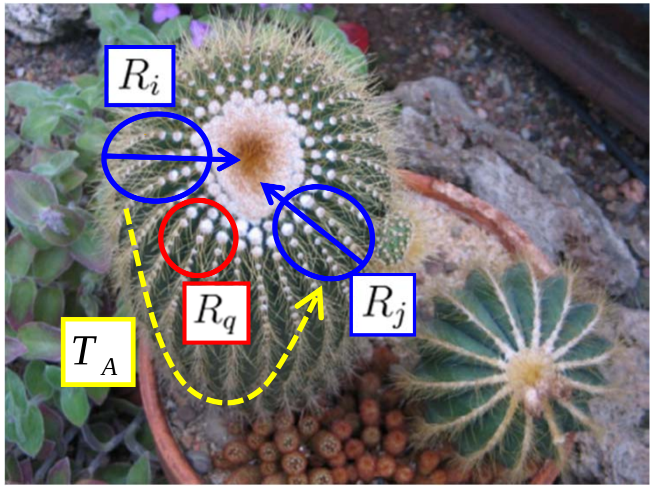
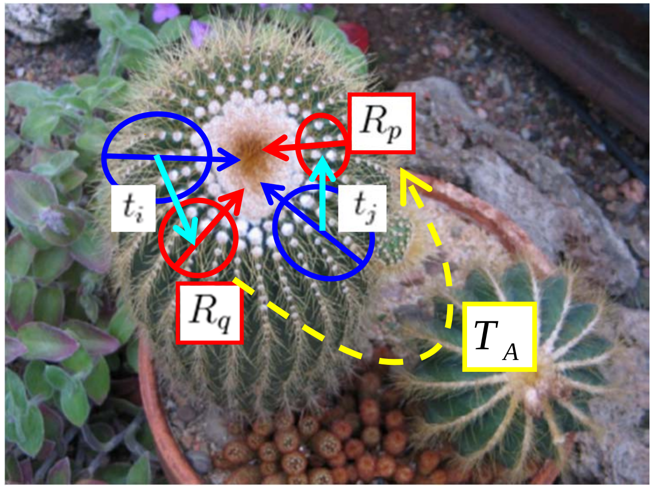
Expansion

- Propagate a neighbor region via a supporter

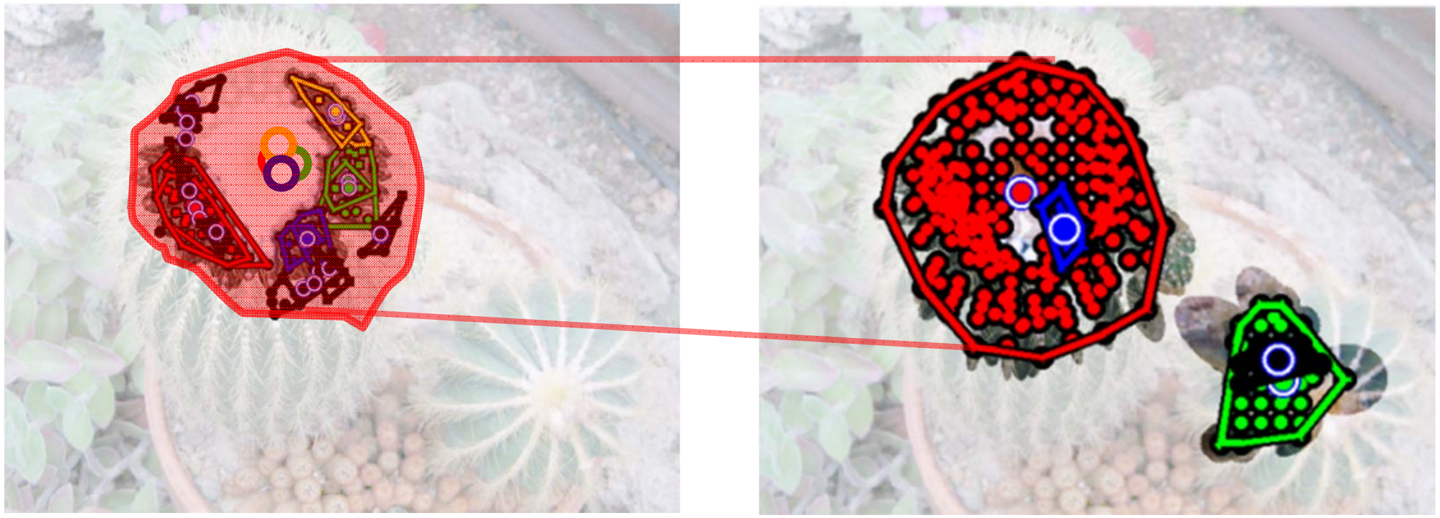




- Supporter: (R_i, R_j), $R_j = T_A R_i$
- Take R_i's nearby region R_p.
- $t_j = T_A t_i$, $R_q = T_A R_p$
- New match generated: (R_p, R_q)
- Expansion layer updated

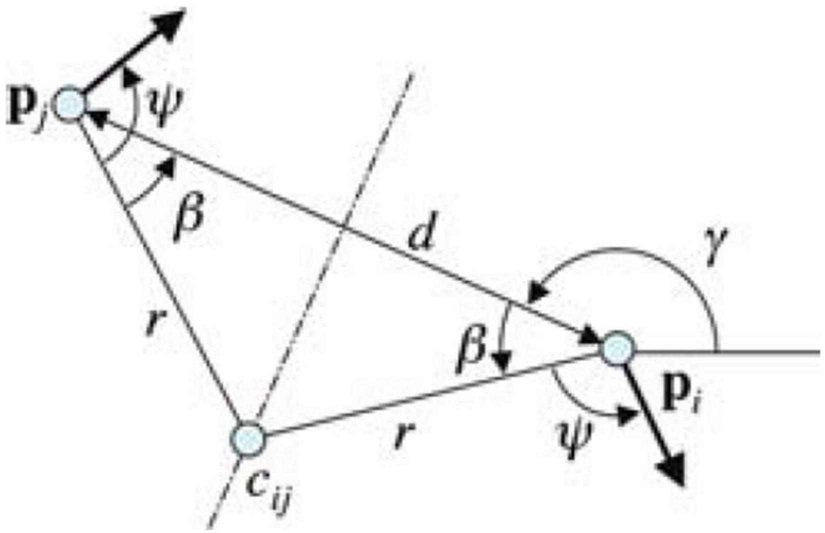
Merge

- Merge when two clusters share the center of rotation
  - Only if a expanded cluster has a *similar center* with another cluster.
  - *similar center*: the distance btw the centers < 10% Image width
  - Expansion layer updated.



Rotational Axis?

- Rotational axis (center) estimation
  - Conventional



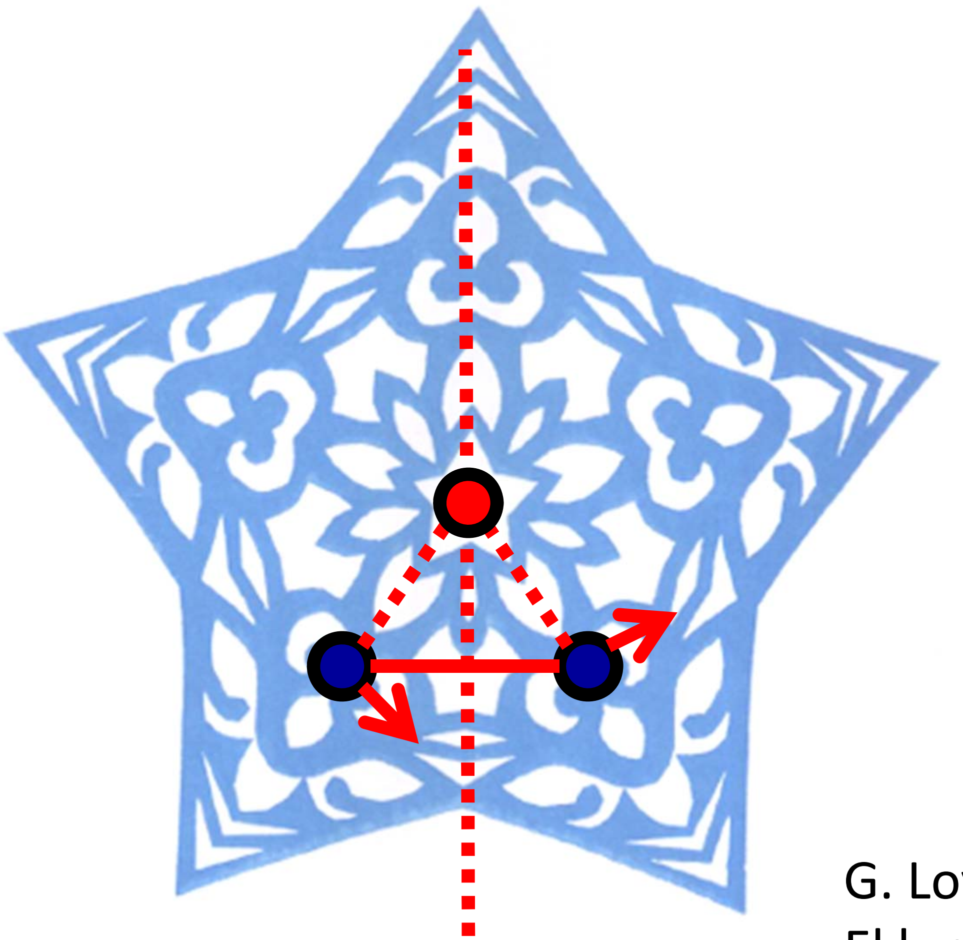
$$c_{ij} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} r\cos(\beta+\gamma) \\ r\sin(\beta+\gamma) \end{bmatrix}$$

$$r^2 = \left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\tan\beta\right)^2$$

$$\phi_i = \gamma + \beta + \psi$$
$$\phi_j = \gamma + \pi - \beta + \psi$$

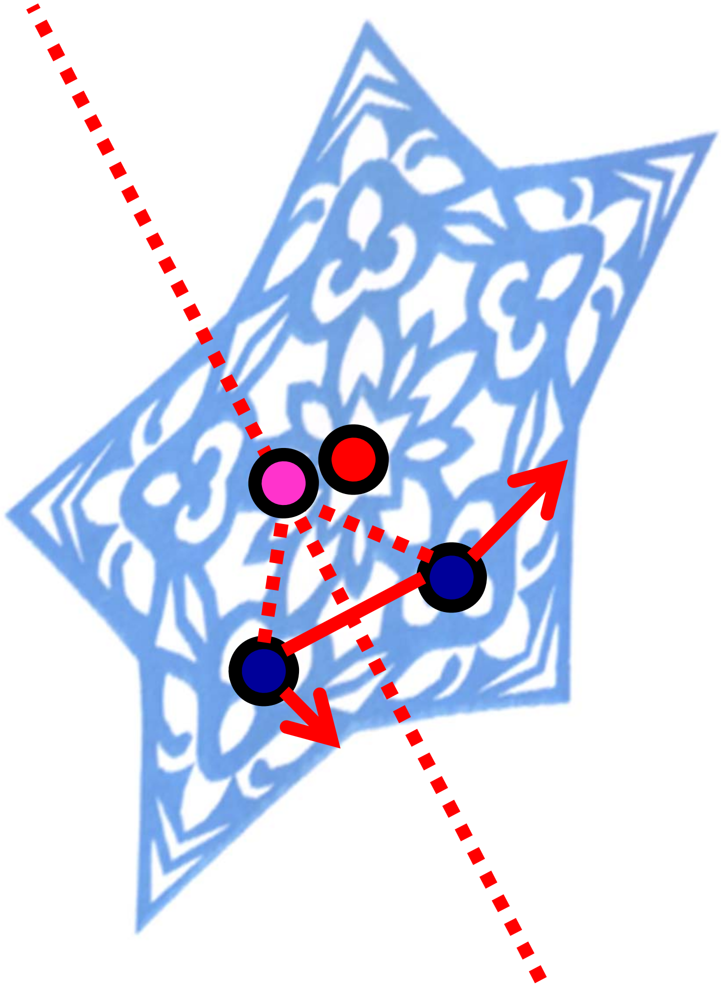
G. Loy and J.O. Eklundh *ECCV2006*

Rotational Axis?



G. Loy and J.O.
Eklundh *ECCV2006*

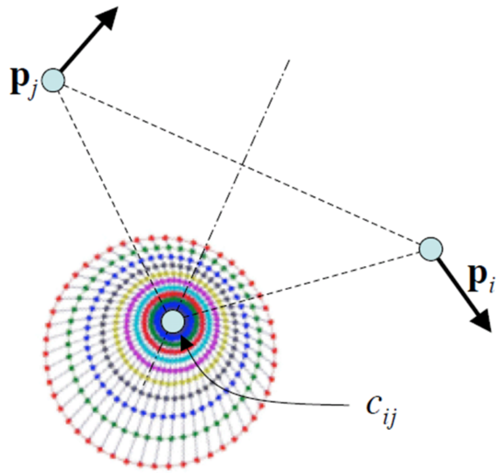
Rotational Axis in Skewed cases



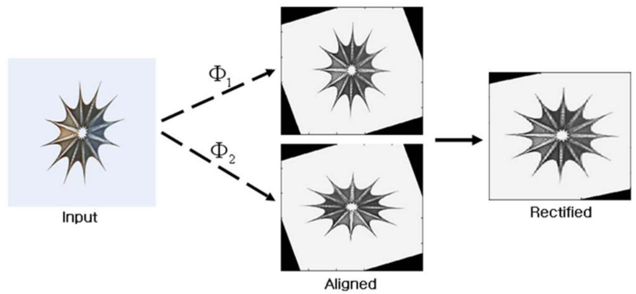
Previous solution to Skewed Cases

- **Local search**

- Cornelius and Loy (ICPR06): compute centers of rotation w.r.t all discretized orientations by tilting angles, then find the most likely center by voting.



- Lee and Liu (PAMI10): use phase analysis of Freize expansion plane, then iteratively rectifying the pattern to find most likely aspect ratio.

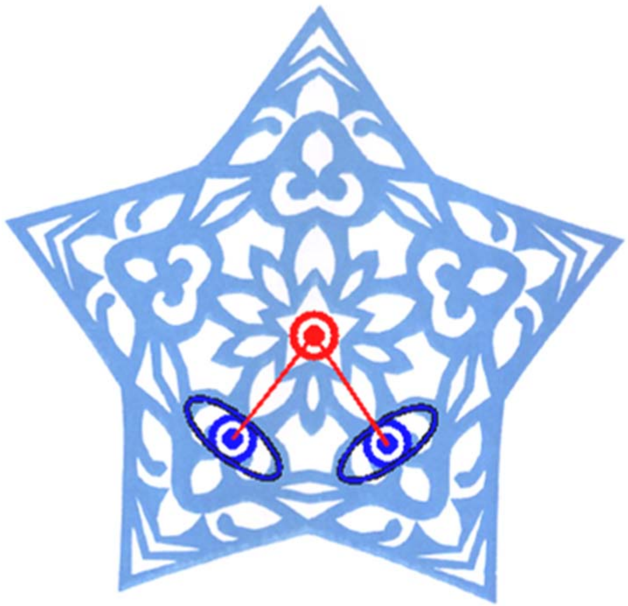


Comparison in Skewed cases

Finding center

Skew Compensated
(Ours)

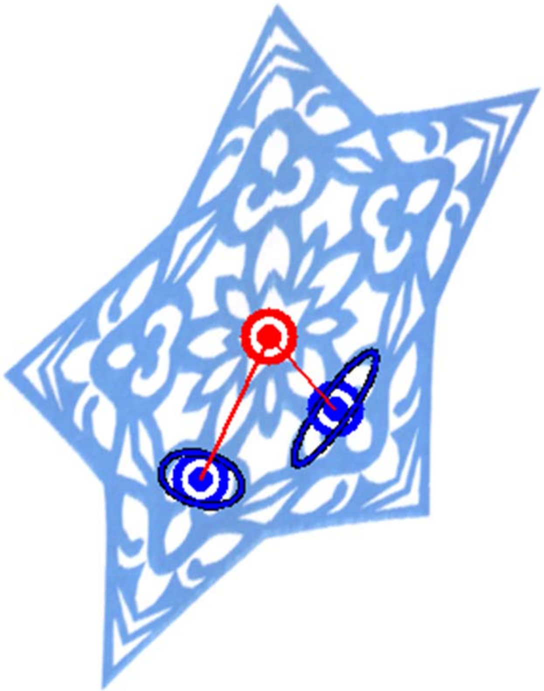
Conventional
(LE06)



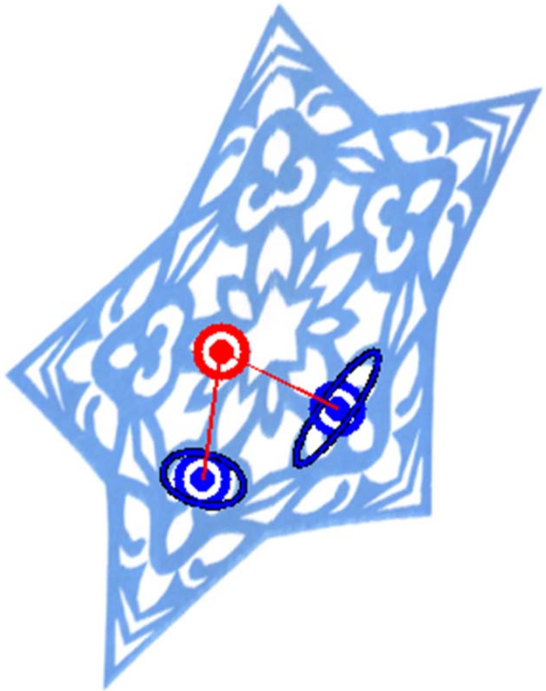
Comparison in Skewed cases

Finding center

Skew Compensated
(Ours)



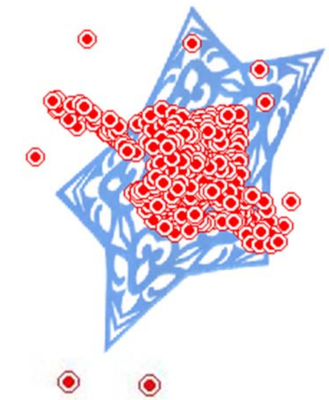
Conventional
(LE06)

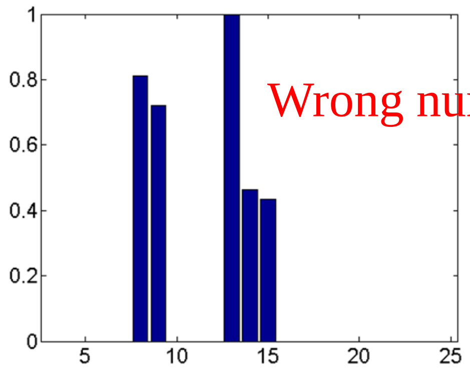


Comparison in Skewed cases

Center Distribution | Angular Histogram

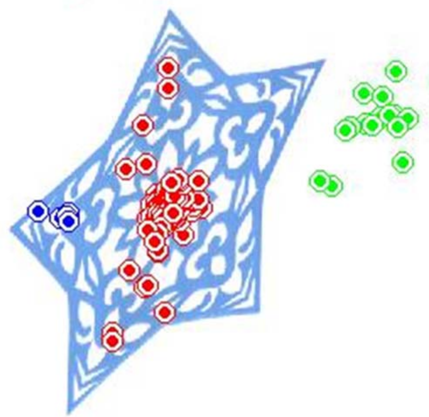
Skew Compensated (Ours)

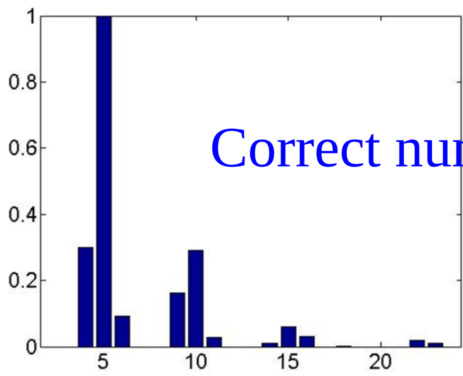




Wrong number of folds: 12

Conventional (LE06)

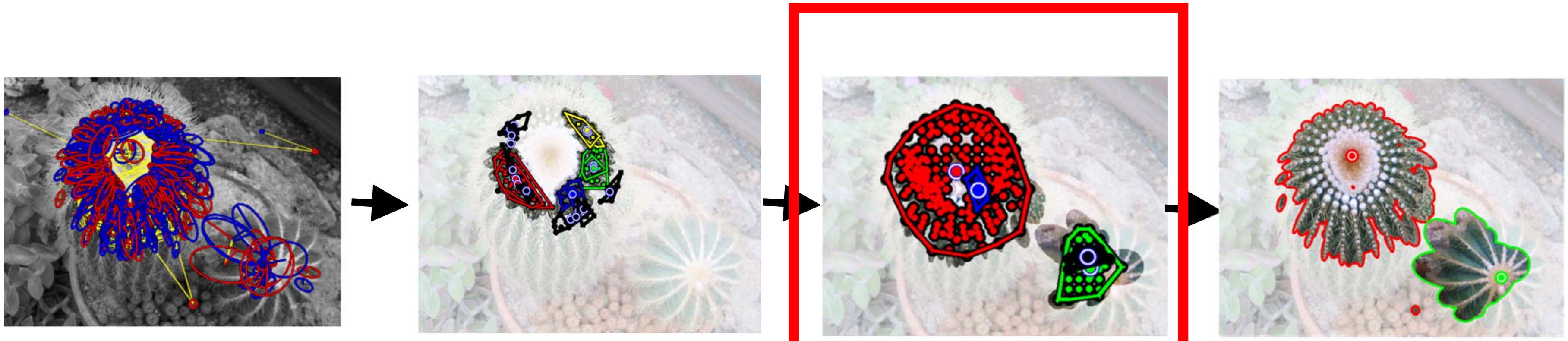




Correct number of folds: 5

Step#3: Symmetry Analysis

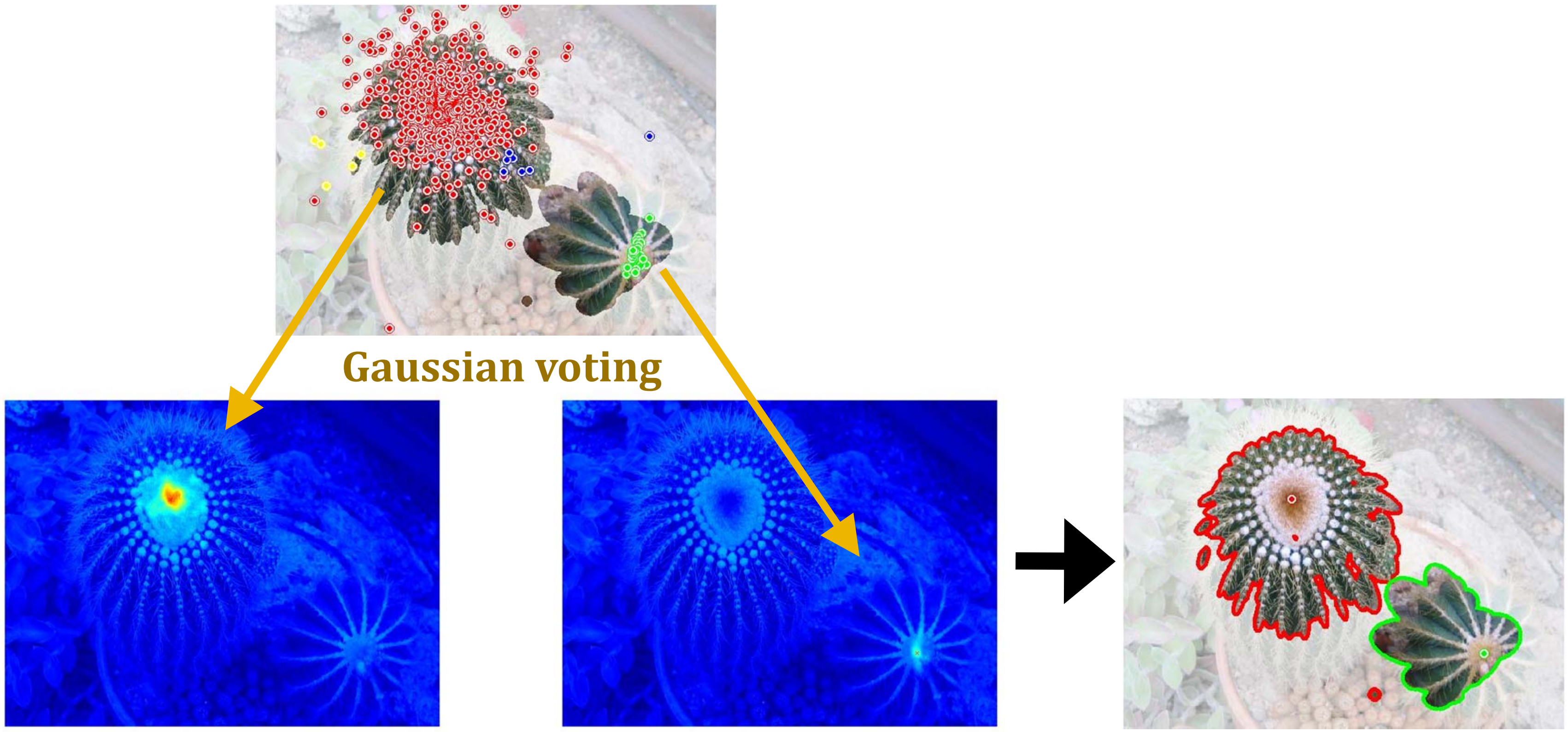
Goal: Estimate the number of folds and the type of detected symmetry



Seed extraction **Symmetry-growing** **Symmetry Analysis** **Verification**

Final Rotational Axis

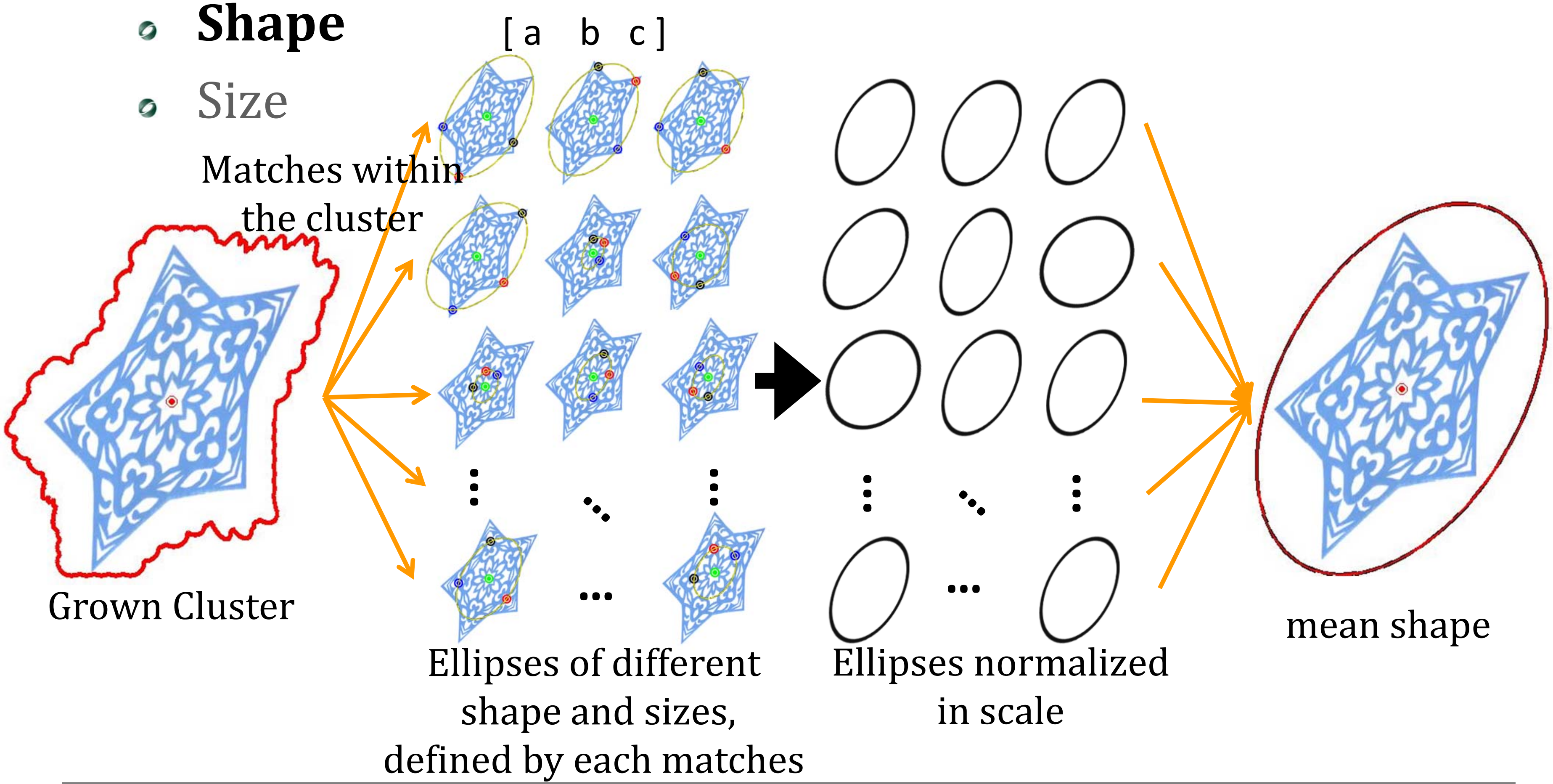
- Gaussian voting



Max. peak selected (cluster 1) | Max. peak selected (cluster 2) | **Result**

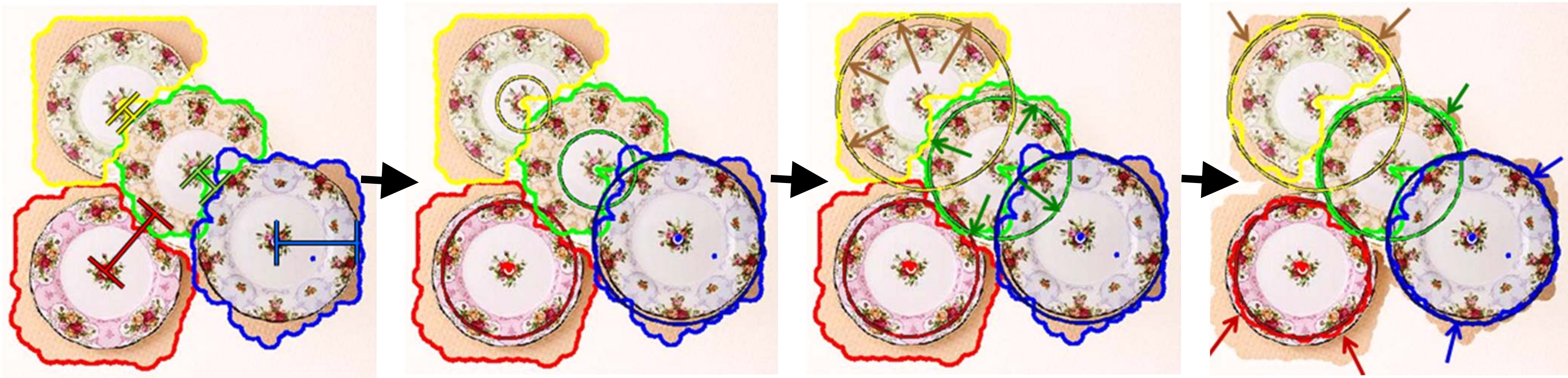
Symmetry Analysis

- Ellipse estimation
  - **Shape**
  - Size



Symmetry Analysis

- Ellipse estimation
  - Shape
  - **Size**



Grown Cluster

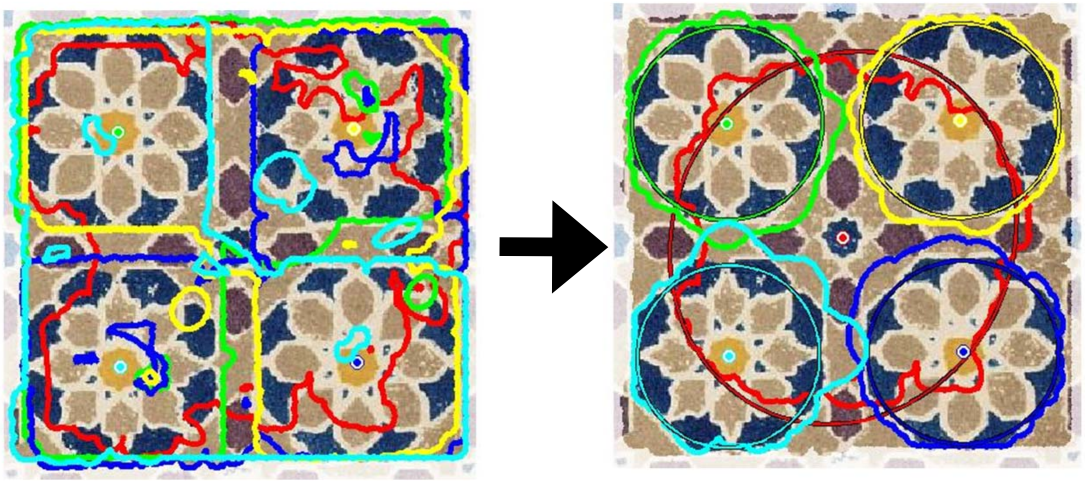
Take min.distance from center to the cluster's convex hull as radius

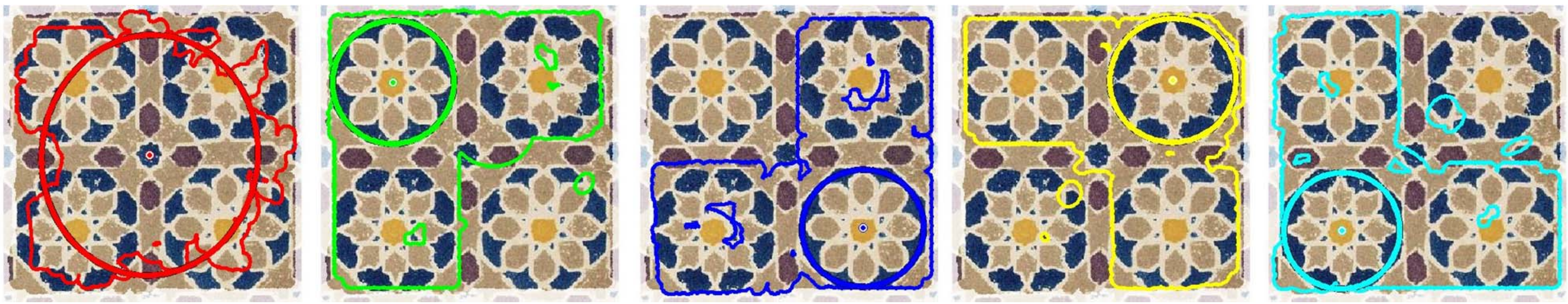
Adaptively increase or decrease the radius in its length

Cluster fitted for the determined ellipse

Symmetry Analysis

- Ellipse estimation
  - Example



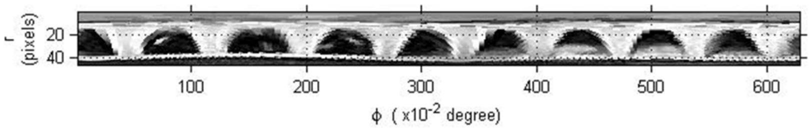


Symmetry Analysis

- Number of folds estimation

Ellipse Estimated

Rectified



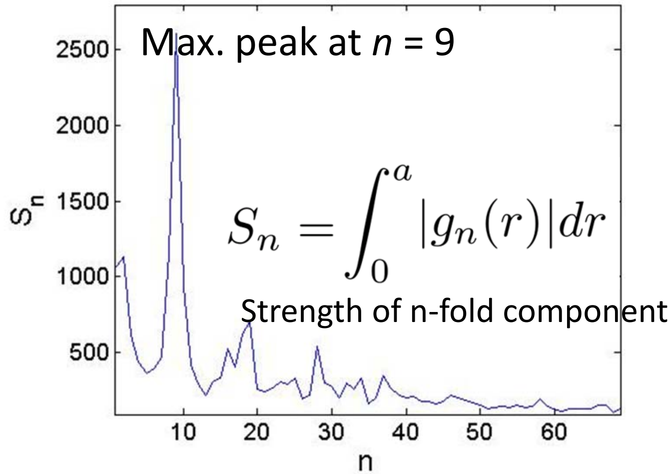
Converted to polar coordinate

$$\rho(r,\phi) = \sum_{n=-\infty}^{\infty} g_n(r) exp(in\phi)$$

Image density

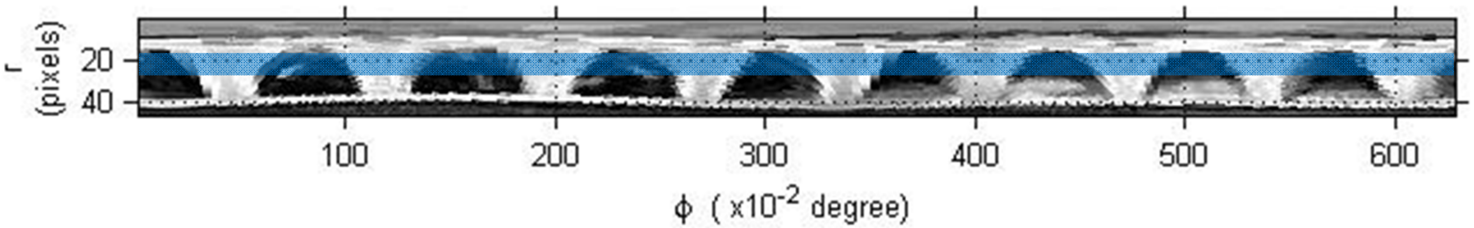
Weight of *n*-fold component

[Crowther and Amos, *JMB 1971*]



Symmetry Analysis

- Symmetry Type Estimation (D_n/C_n)



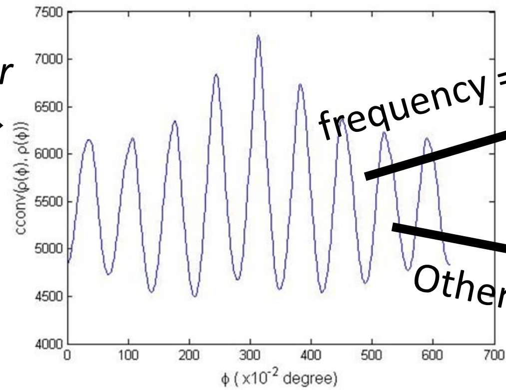
$\rho(r, \phi)$

$\circledast$

$\rho(r, \phi)$

Circular Convolution with itself
= Mirror and correlation

Integrate along *r*



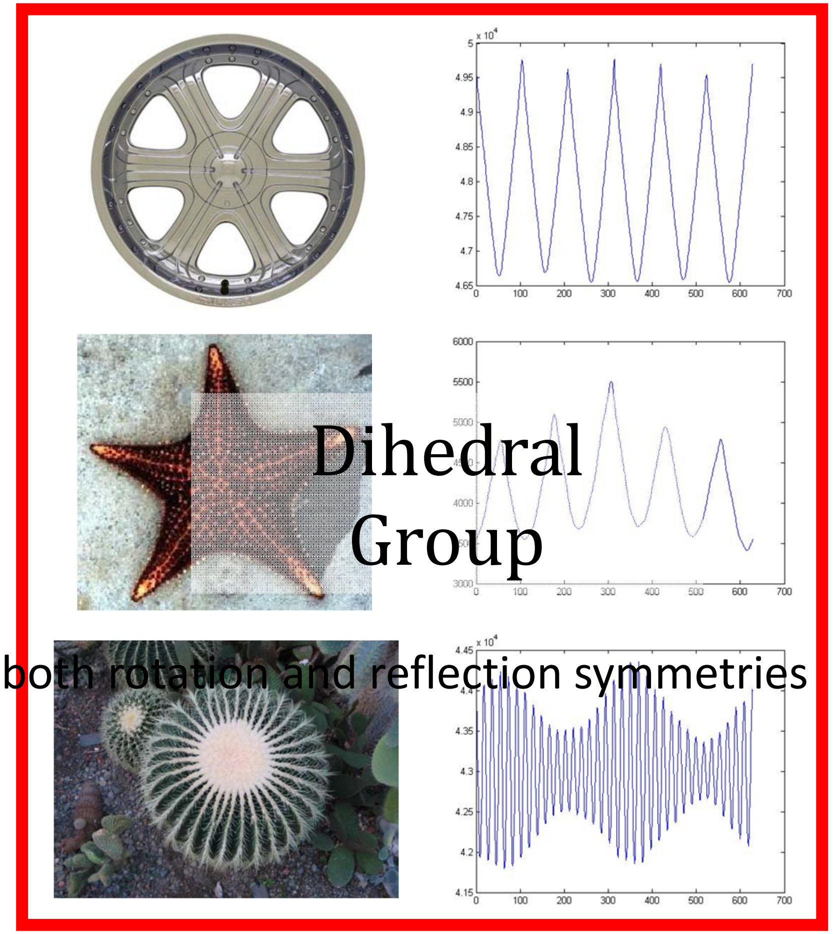
frequency = # of folds → D_n Dihedral

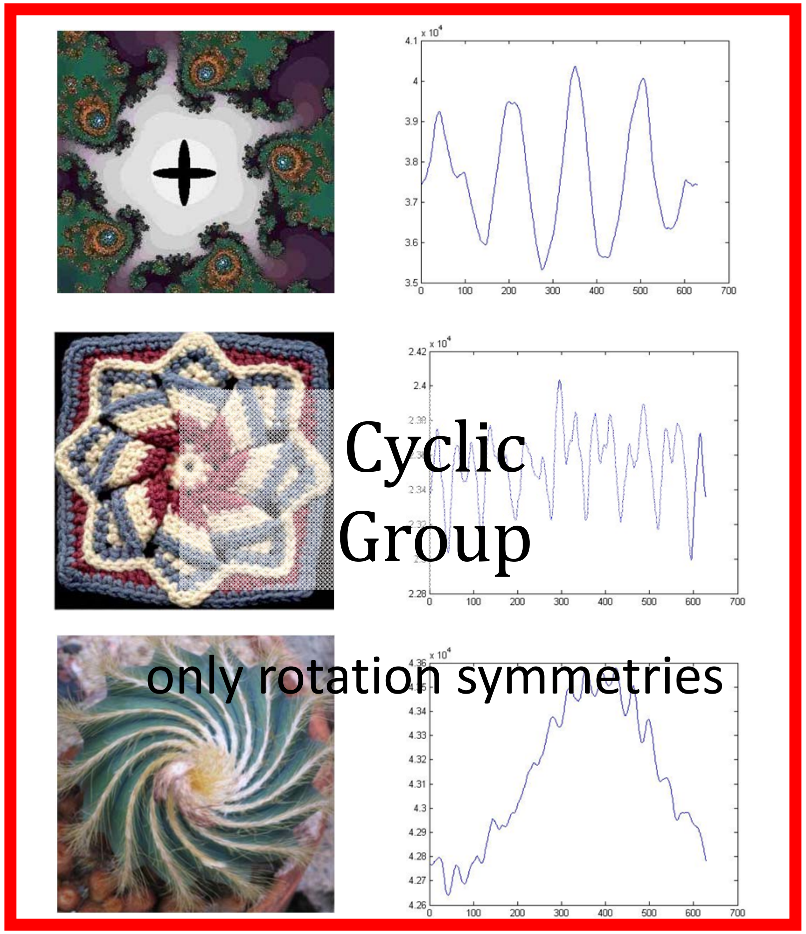
Otherwise → C_n Cyclic

[Similar to the work of S. Lee and Y. Liu CVPR2008]

Symmetry Analysis

- Symmetry Type Estimation (D_n/C_n)





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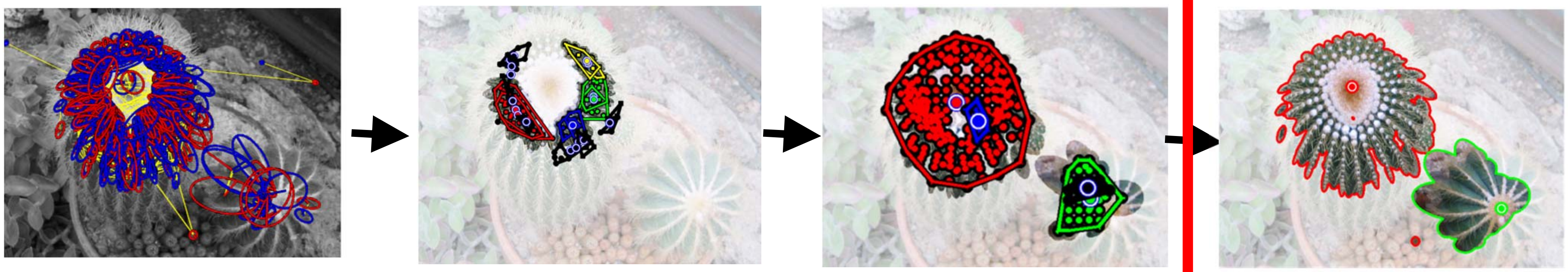
Step#4: Symmetry Verification

Goal: Eliminate the unreliable clusters from the grown symmetry clusters



Seed extraction **Symmetry-growing** **Symmetry Analysis** **Verification**

Symmetry Cluster Verification

- Discard symmetries with trivial # of folds (1 or 2)
- Remove symmetry having high center variance

Experimental Results

- Settings
  - MSER & Hessian affine detector, SIFT descriptor
  - Parameters
    - Radius of latent regions r_a 1/25 * the shorter image axis
    - Cluster size threshold δ_a 0.02
    - Center variance threshold δ_b 3 * the dominant cluster's variance

- Test dataset
  - The dataset of S. Lee and Y. Liu's PAMI 2010 work.

Quantitative Evaluation

- Used evaluation result in Lee and Liu's PAMI 2010 work

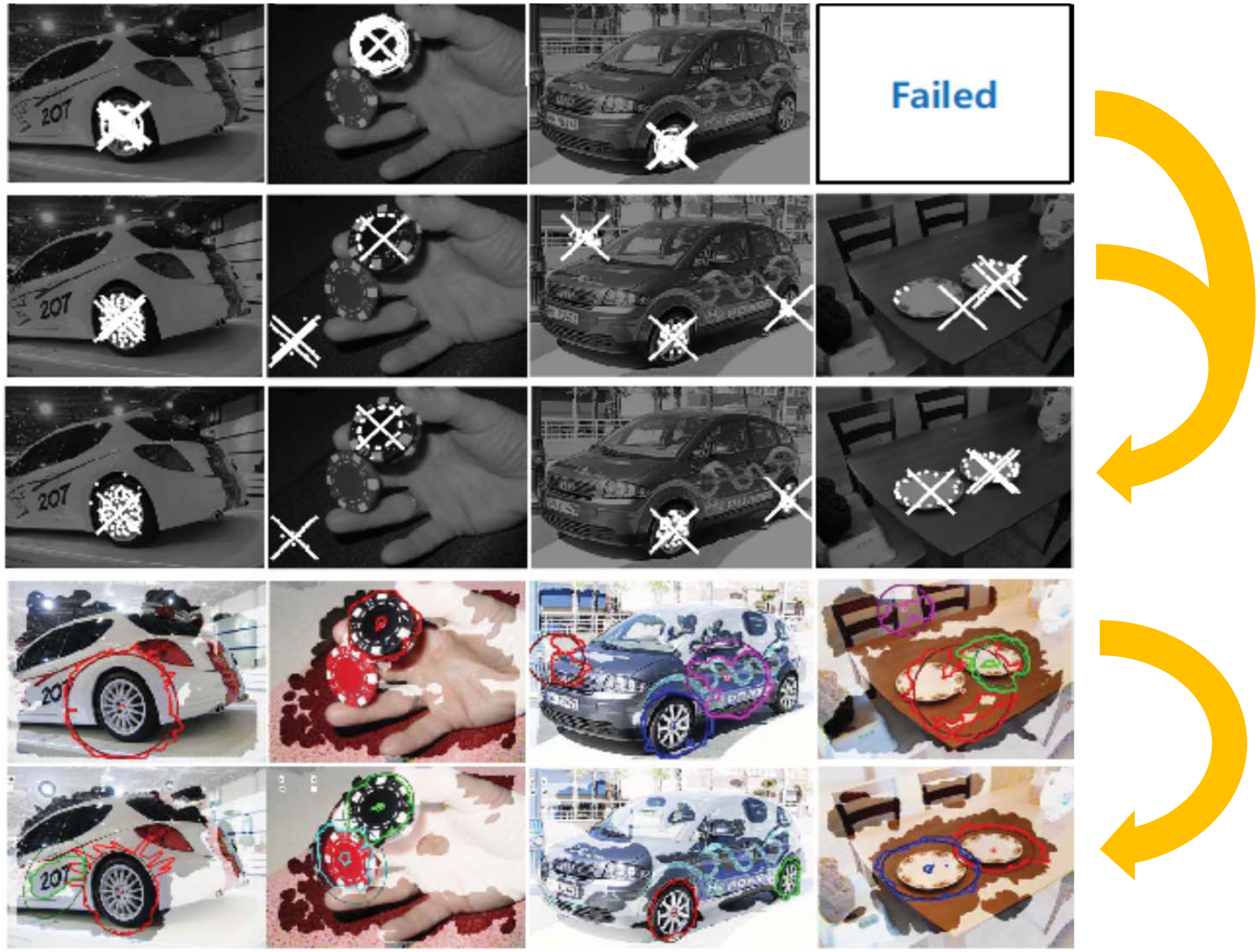
Algorithm	Data Set	TP Center Rate	FP Center Rate	# of Folds	Symmetry Type
Loy and Eklundh ECCV 2006	Synthetic (29 images/48 GT)	31/48 = 65 %	4/48 = 8%	22/49 = 45%	N/A
	Real-Single (58 images/58 GT)	50/58 = 86%	41/58 = 71%	16/64 = 25%	N/A
	Real-Multi (21 images/78 GT)	32/78 = 41%	6/78 = 8%	12/42 = 29%	N/A
	Overall(108 images/184 GT)	113/184 = 61%	51/184 = 28%	50/155 = 32%	N/A
Lee and Liu CVPR 2008	Synthetic (29 images/48 GT)	36/48 = 75 %	0/48 = 0%	42/54 = 78%	44/54 = 81%
	Real-Single (58 images/58 GT)	25/58 = 43%	33/58 = 57%	22/32 = 69%	24/32 = 75%
	Real-Multi (21 images/78 GT)	19/78 = 24%	21/78 = 27%	18/25 = 72%	19/25 = 76%
	Overall(108 images/184 GT)	80/184 = 43%	54/184 = 29%	82/111 = 74%	87/111 = 78%
Lee and Liu PAMI 2010 # 2	Synthetic (29 images/48 GT)	43/48 = 90 %	12/48 = 25%	44/62 = 71%	51/62 = 82%
	Real-Single (58 images/58 GT)	54/58 = 93%	31/58 = 53 %	35/66 = 53%	54/66 = 82%
	Real-Multi (21 images/78 GT)	55/78 = 71%	22/78 = 28%	40/70 = 57%	53/70 = 76%
	Overall(108 images/184 GT)	152/184 = 83%	65/184 = 35%	119/198 = 60%	158/198 = 80%
Ours	Synthetic (29 images/48 GT)	43/48 = 90 %	11/48 = 23%	34/48 = 71%	23/48 = 48%
	Real-Single (58 images/58 GT)	55/58 = 95%	25/58 = 43%	29/58 = 50%	41/58 = 71%
	Real-Multi (21 images/78 GT)	55/78 = 71%	21/78 = 26%	48/78 = 62%	46/67 = 69%
	Overall(108 images/184 GT)	153/184 = 83%	57/184 = 31%	111/184 = 60%	110/173 = 64%

- Low FP rate while comparable or higher TP rate
- Competitive result for #folds detection
- However, poor performance in rotation type estimation

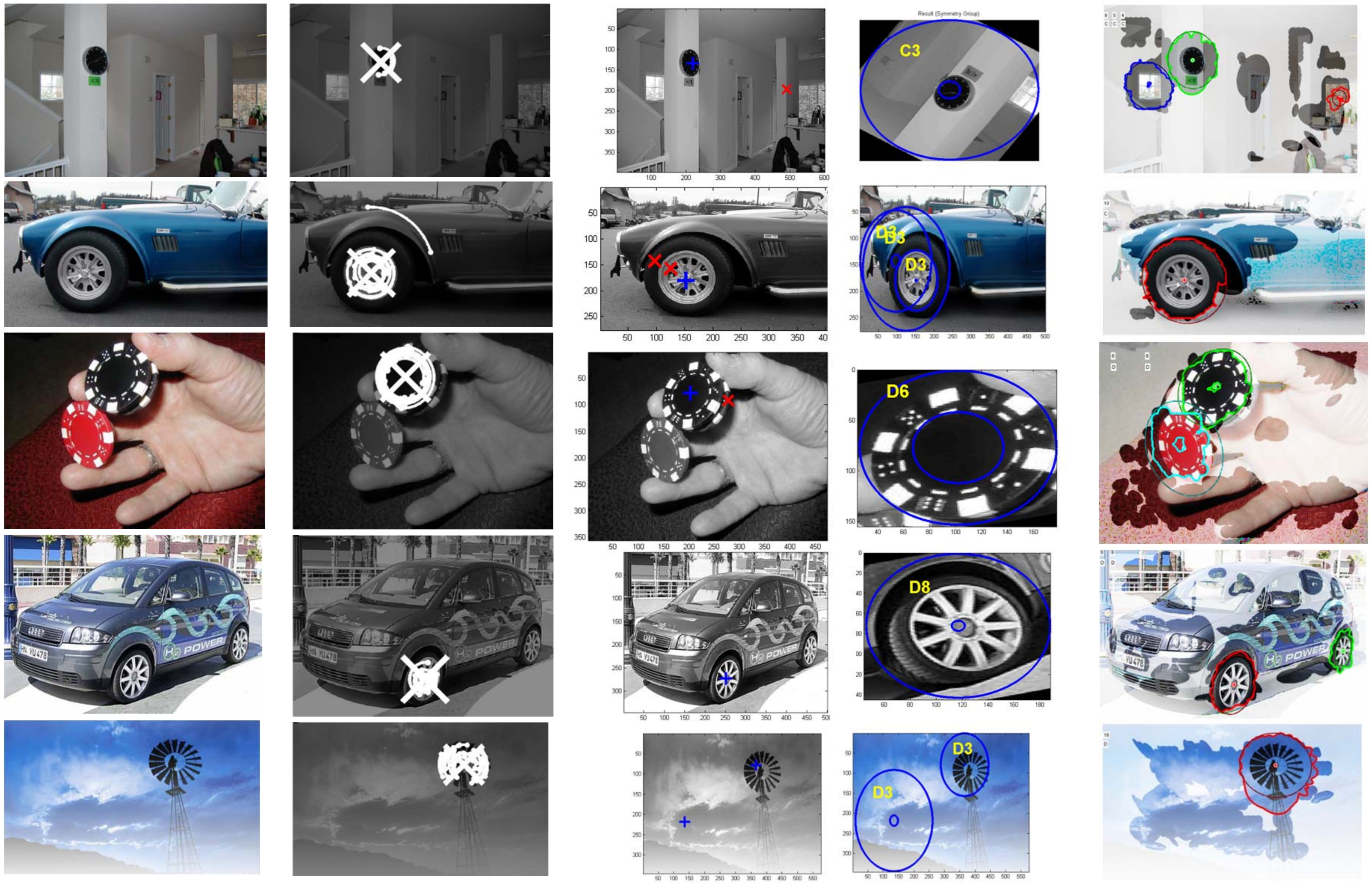
Effect of Skew-robust estimation

- Our results on images with single symmetry patterns

LE06	
LE06 +affCovFeats	
LE06 +affCovFeats *+Skew-robust axis estimation*	
Symmetry Growing Only	
Ours (SymmetryGrow *+Skew-robust axis estimation*)	

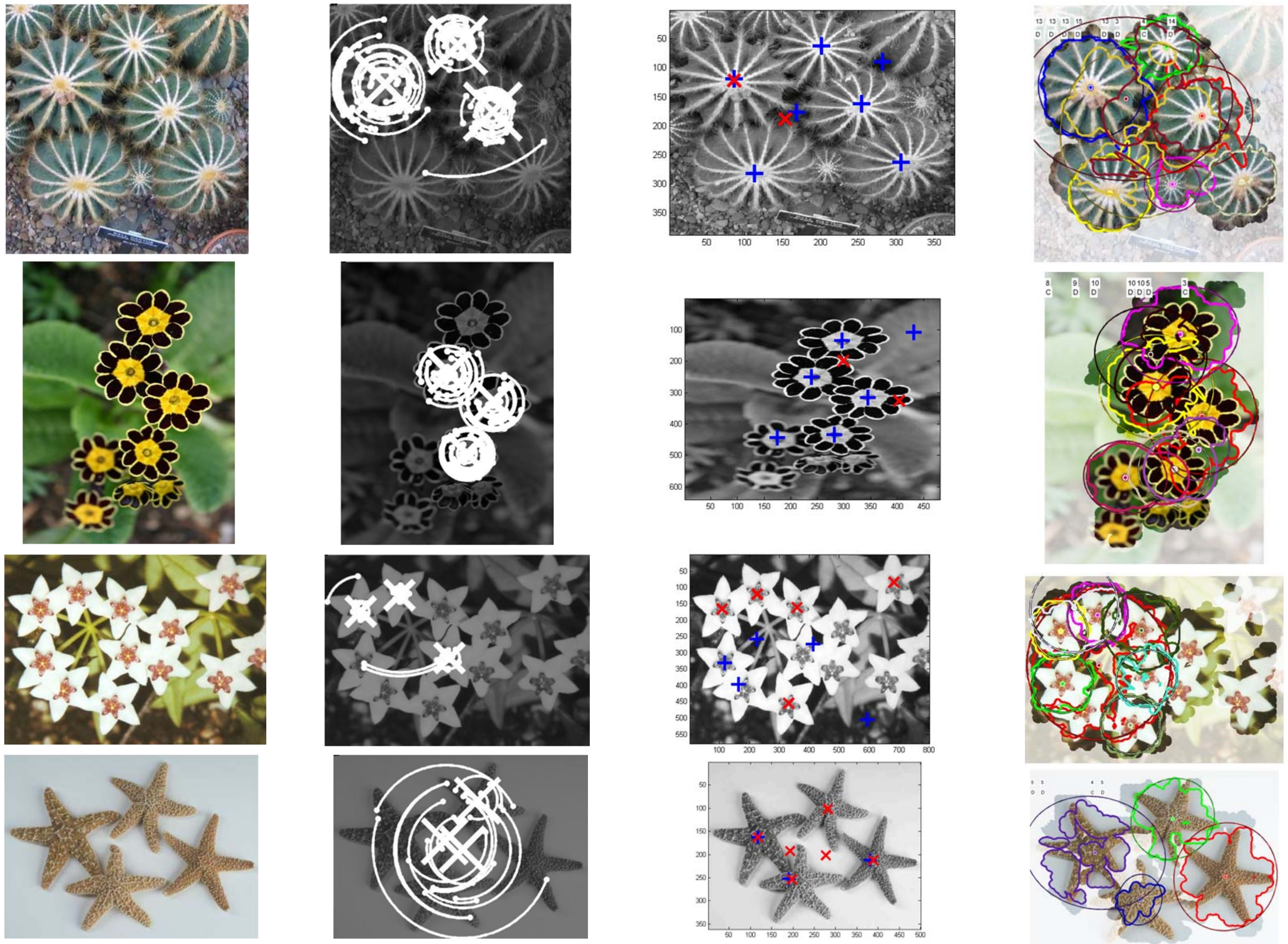


Comparative examples (1/3)



Input **LE06** **LL10** **Our Result**

Comparative examples (2/3)



Input **LE06** **LL10** **Our Result**

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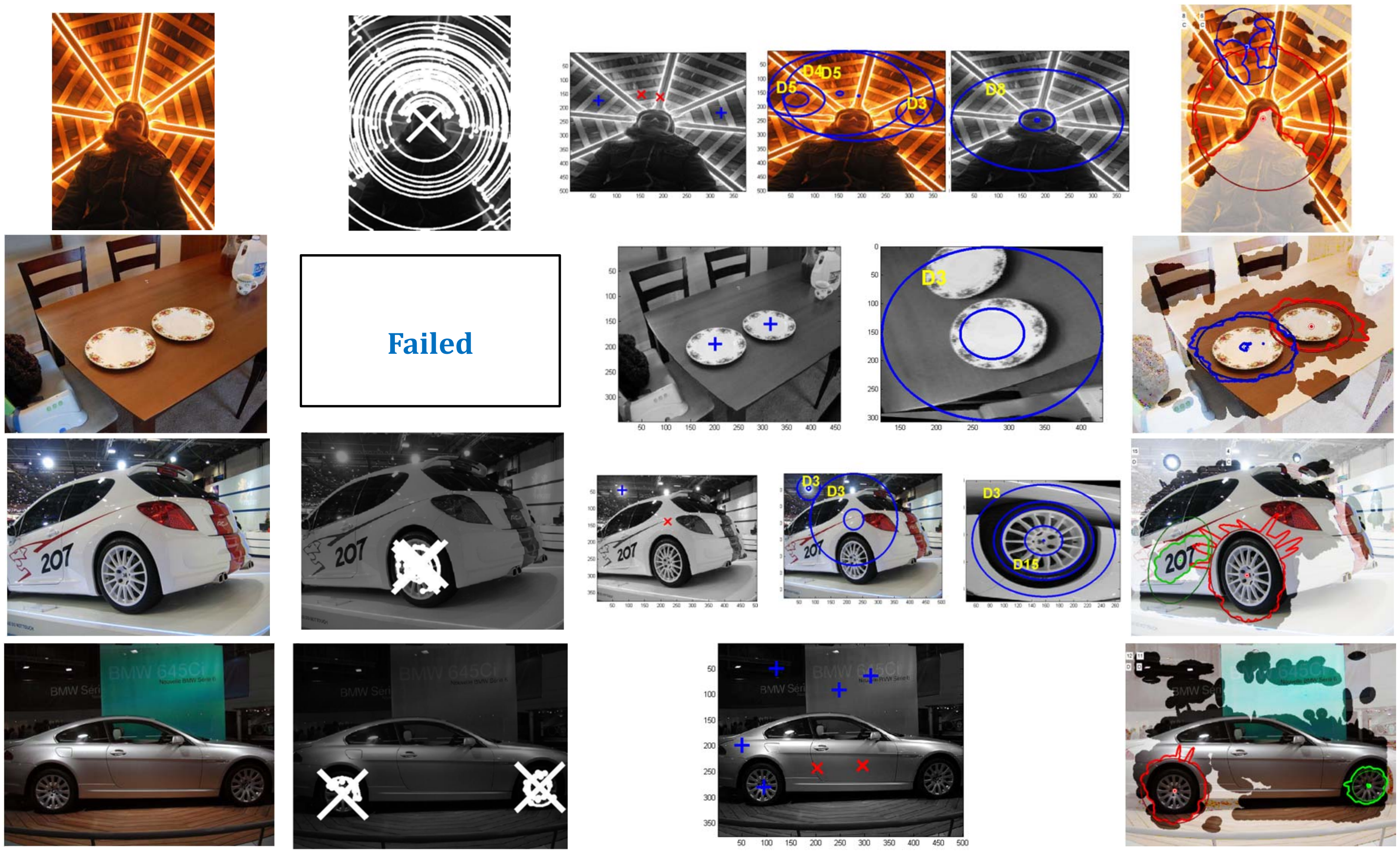
Comparative examples (3/3)



Input LE06 LL10 Our Result

Conclusion & Future Work

- Symmetry-Growing
  - overcomes the locality of local feature based methods
  - detects detailed partial symmetries
- Skew-Robust Axis Estimation
  - robust to affine deformation (or others)
  - fast and closed-form solution to skewed symmetry
- Future Work
  - Large deformation in symmetry
  - Effective growing strategy for each symmetry type

Thanks for your attention!

http://cv.snu.ac.kr